CHPTER EIGHT

Flow of Compressible Fluid

8.1 Introduction

All fluids are to some degree compressible, compressibility is sufficiently great to affect flow under normal conditions only for a ***gas***. If the pressure of the gas does not change by more than about 20%, [or when the change in density more than 5-10 %] it is usually satisfactory to treat the gas as incompressible fluid with a density equal to that at the mean pressure.

When compressibility is taken into account, the equations of flow become more complex than they are for an incompressible fluid.

The flow of gases through orifices, nozzles, and to flow in pipelines presents in all these cases, the flow may reach a limiting maximum value which independent of the downstream pressure (P2); this is a phenomenon which does not arise with incompressible fluids.

8.2 Velocity of Propagation of a Pressure Wave

The velocity of propagation is a function of *the bulk modulus of elasticity (ε), where;*

$$\varepsilon = \frac{\text{increase of stress within the fluid}}{\text{resulting volumetric strain}} = \frac{dP}{-dv/v}$$

$$\therefore \varepsilon = -v\frac{dP}{dv}$$

where, υ: specific volume (υ = 1/ρ).

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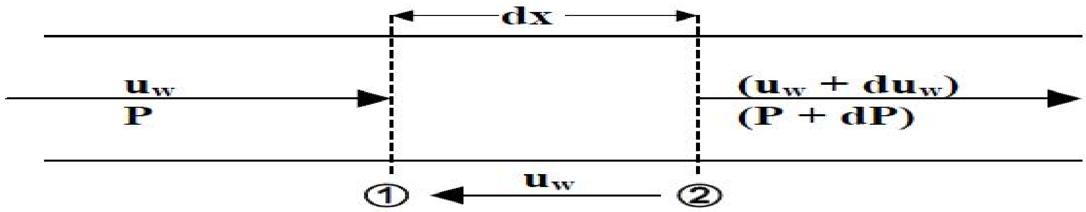
Suppose a pressure wave to be transmitted at a velocity uw over a distance dx in a fluid of cross-sectional area A, from section ❶ to section ❷ as shown in Figure;



Now imagine the pressure wave to be brought to rest by causing the fluid to flow at a velocity u_w *<u>in the opposite direction.</u>*

From conservation of mass law;

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \rho u_w A = (\rho + d\rho)(u_w + du_w)A$$

$$\rightarrow \frac{u_w}{v} A = \frac{u_w + du_w}{(v + dv)} A$$

$$and\ \dot{m} = \frac{u_w}{v} A \rightarrow\rightarrow u_w = \frac{\dot{m}}{A} \rightarrow\rightarrow du_w = \frac{\dot{m}}{A} dv$$

Newton's 2[nd] law of motion stated that "The rate of change in momentum of fluid is equal to the net force acting on the fluid between sections ❶ and ❷ .

Thus;

$$\dot{m}[(u_w - du_w) - u_w] = A[P - (P + dP)] \rightarrow\rightarrow \frac{\dot{m}}{A} du_w = -dP$$

$$but\ du_w = \frac{\dot{m}}{A} dv \rightarrow \frac{\dot{m}}{A}\left(\frac{\dot{m}}{A} dv\right) = -dP \rightarrow \frac{-dP}{dv} = \left(\frac{\dot{m}}{A}\right)^2$$

$$we\ have\quad \frac{-dP}{dv} = \frac{\varepsilon}{v} \rightarrow\rightarrow \frac{\varepsilon}{v} = \left(\frac{\dot{m}}{A}\right)^2 = G^2$$

$$\frac{\varepsilon}{v} = \left(\frac{u_w A/v}{A}\right)^2 = (u_w/v)^2 \rightarrow\rightarrow u_w^2 = v\,\varepsilon \qquad \boxed{\therefore u_w = \sqrt{v\,\varepsilon}}$$

For ideal gases

$$Pv^k = constant \qquad where,\ k = 1.0 \text{ for isother}$$

$$k = \gamma \quad \text{for isentropic conditions}, \gamma = \frac{c_p}{c_v}$$

$$d\left(Pv^k\right) = 0.0 \rightarrow\ v^k dp + Pkv^{k-1}dv = 0$$

$$v^k dP = -kP\frac{v^k}{v}dv \rightarrow \frac{dP}{dv} = -K\frac{P}{v} \rightarrow -v\left(\frac{dP}{dv}\right) = kP = \varepsilon$$

$$\therefore u_w = \sqrt{kPv}$$

$$-\text{For isothermal conditions k} = 1 \rightarrow \therefore u_w = \sqrt{Pv}$$

$$-\text{For isentropic (adiabatic) conditions k} = \gamma \rightarrow \therefore u_w = \sqrt{\gamma P v}$$

The value of $\mathbf{u_w}$ is found to correspond closely to **<u>the velocity of sound in the fluid</u>** and its correspond to the velocity of the fluid at the end of a pipe uder conditions of maximum flow.

<u>Mach Number</u>

Is the ratio between gas velocity to sonic velocity,

$$M_a = \frac{u}{u_w}$$

where,

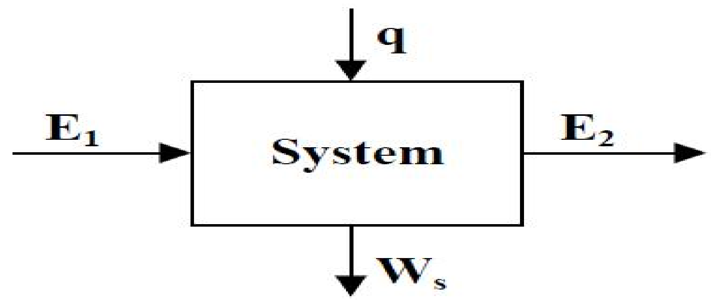
Ma > 1 supersonic velocity; Ma = 1 sonic velocity ; Ma < 1 subsonic velocity

8.3 General Energy Equation for Compressible Fluids

Let E the total energy per unit mass of the fluid where,

E=Internal energy (U)+Pressure energy (Pυ)+Potential nergy(zg)+Kinetic energy ($u^{2/}2$)

Assume the system in the Figure;



Energy balance

$\mathbf{E_1 + q = E_2 + Ws \Rightarrow E2 - E1 = q - Ws}$

$\mathbf{\Rightarrow \Delta U + \Delta(P\upsilon) + g\Delta(z) + \Delta(u^2/2) = q - Ws}$

[α = 1 for compressible fluid since it almost in turbulent flow]

but $\Delta H = \Delta U + \Delta(P\upsilon)$

$\Rightarrow \Delta H + g\Delta(z) + \Delta(u^2/2) = q - Ws$

$dH + gd(z) + ud(u) = dq - dWs$

but,

$\mathbf{dH = dq + dF + \upsilon dP}$

$dH = dq + dF + \upsilon dP$

Comes from:-

For irreversible process

$dW = Pd\upsilon - dF$ -------useful work

$dU = dq - dW$ -------closed system

$dH = dU + d(P\upsilon)$

$= dq - dW + d(P\upsilon)$

$= dq - (Pd\upsilon - dF) + d(P\upsilon)$

$= dq - Pd\upsilon + dF + Pd\upsilon + \upsilon dP$

$\Rightarrow \mathbf{dH = dq + dF + \upsilon dP}$

where,

dF: amount of mechanical energy converted into heat

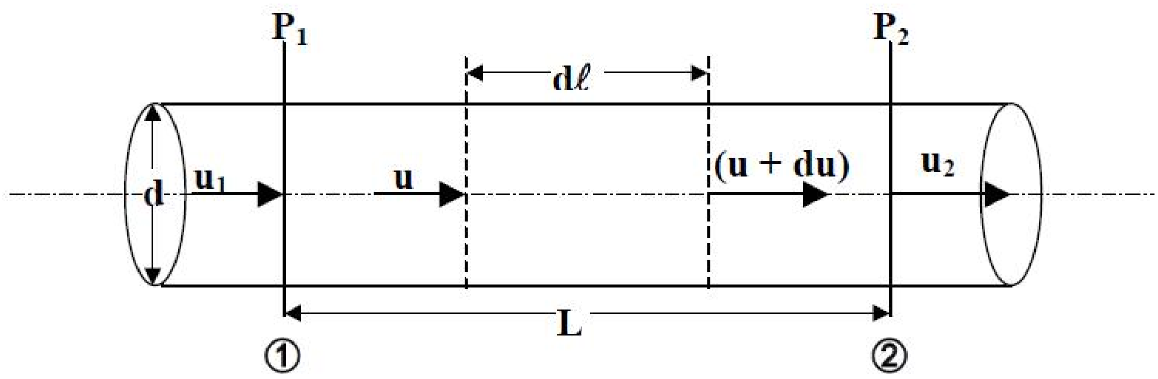
$\mathbf{\Rightarrow u\,du + g\,dz + \upsilon\,dP + dWs + dF = 0}$

$$\frac{\Delta u^2}{2} + g\Delta z + \int_{p_1}^{p_2} v\,dP + Ws + F = 0$$

General equation of energy apply
To any type of fluid

●For compressible fluid flowing through (dl) of pipe of constant area

$u\,du + g\,dz + \upsilon\,dP + dW_s + 4\Phi\,(dl/d)\,u^2 = 0$ --------------(*)



$$\dot{m}\,\rho\,u\,A \rightarrow \rightarrow \frac{\dot{m}}{A} = \frac{u}{v} = G \qquad \therefore u = G\,v \rightarrow \rightarrow du = G\,dv$$

Substitute these equations into equation (*), to give

$G\,\upsilon\,(G\,d\upsilon) + g\,dz + \upsilon\,dP + dW_s + 4\Phi\,(dl/d)\,(G\,\upsilon)^2 = 0$

For horizontal pipe (dz = 0), and no shaft work (Ws =0)

$\Rightarrow G^2\,\upsilon\,(d\upsilon) + \upsilon\,dP + 4\Phi\,(dl/d)\,(G\,\upsilon)^2 = 0$ ---------------------------(**)

Dividing by (υ2) and integrating over a length L of pipe to give;

$$G^2 \ln\left(\frac{v_2}{v_1}\right) + \int_{p_1}^{p_2} \frac{dP}{v} + 4\ \Phi \frac{L}{d} G^2 = 0$$

Kinetic Energy **Pressure energy** **Frictional energy**

General equation of energy apply tocompressible fluid in horizontal pipe withno shaft work

8.3.1 Isothermal Flow of an Ideal Gas in a Horizontal Pipe

For isothermal conditions of an ideal gas

$P\,\upsilon = \text{constant} \Rightarrow P\,\upsilon = P_1\,\upsilon_1 \Rightarrow 1/\upsilon = P / (P_1\,\upsilon_1)$

$$\int_{p_1}^{p_2}\frac{dP}{v}=\frac{1}{P_1v_1}\int_{p_1}^{P_2}P\,dP=\frac{1}{2P_1v_1}\left(P_2^2-p_1^2\right)----(1)$$

$\mathbf{P_1\, \upsilon_1 = P_2\, \upsilon_2 \Rightarrow \upsilon_2 / \upsilon_1 = P_1/ P_2}$ **-----------------------(2)**

Substitute equations (1) and (2) into the genral equation of compressibl fluid to give;

$$G^2 \ln\left(\frac{P_1}{P_2}\right)+\frac{\left(P_2^2-p_1^2\right)}{2P_1v_1}+4\ \phi\frac{L}{d}G^2=0$$

Let υ_m the mean specific volume at mean pressure Pm, where,

$\mathbf{P_m = (P_1 + P_2)/2}$

$\mathbf{P_m\, \upsilon_m = P_1\, \upsilon_1 \Rightarrow P_m = (P_1 + P_2)/2 = P_1\, \upsilon_1/ \upsilon_m}$

$$\frac{\left(P_2^2-p_1^2\right)}{2P_1v_1}=\left(\frac{P_2+P_1}{2}\right)\left(\frac{P_2-P_1}{2}\right)=\left(\frac{P_1v_1}{v_m}\right)\left(\frac{P_2-P_1}{P_1v_1 2}\right)$$

$$\therefore\ \frac{P_2^2-p_1^2}{2P_1v_1}=\frac{P_2-P_1}{v_m}$$

$$G^2 \ln\left(\frac{P_1}{P_2}\right)+\frac{P_2-P_1}{v_m}+4\ \phi\frac{L}{d}G^2-0$$

If $(P_1 - P_2) / P_1 < 0.2$ the fist term of kinetic energy $[G^2 \ln(P_1/P_2)]$ is negligible.

$$-\Delta P=\left(P_1-P_2\right)=4\ \phi\frac{L}{d}G^2 v_m=+4\ \phi\frac{L}{d}\rho_m u_m^2=4f\frac{l}{d}\frac{\rho_m u_m^2}{2}$$

It is used for low-pressure drop.

(i.e. the fluid can be treated as an incompressible fluid at the mean pressure in the pipe.)

8.3.1.1 Maximum Velocity in Isothermal Flow

From equation of isothermal conditions,

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - p_1^2)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$

the mass velocity G = 0 when ($P_1 = P_2$)

At some intermediate value of P2, the flow must therefore be a maximum. To find it, the differentiating the above equation with respect to P_2 for constant P1 must be obtained.

i.e. ($dG/dP_2 = 0$),

First dividing the above equation by G^2

$$\frac{(P_2^2 - p_1^2)}{2P_1 v_1}\frac{1}{G^2} + \ln\left(\frac{P_1}{P_2}\right) + 4\ \phi \frac{L}{d} = 0$$

Then differentiating with respect to P_2

$$\frac{2P_2}{2P_1 v_1 G^2} + \frac{(P_2^2 - p_1^2)}{2P_1 v_1}(-2G^{-3})\frac{dG}{dP_2} + \frac{1}{P_1/P_2}\left(\frac{-P_1}{P_2^2}\right) = 0$$

Rearrangement

$$\frac{P_2}{P_1 v_1 G^2} + \frac{2}{G^3}\left(\frac{(P_2^2 - p_1^2)}{2P_1 v_1}\right)\frac{dG}{dP_2} - \frac{1}{P_2} = 0$$

maximum velocity when ($dG/dP_2 = 0$) where, $P_2 = P_w$, and $G = G_w$

$$\frac{P_w}{P_1 v_1 G_w^2} = \frac{1}{P_w} \Rightarrow G_w^2 = \frac{P_w^2}{P_1 v_1}$$

but for isothermal conditions $P_1 \upsilon_1 = P_w \upsilon_w \Rightarrow P_w = P_1 \upsilon_1 / \upsilon_w$

$$\Rightarrow G_w^2 = \frac{P_w}{P_1 v_1} \Rightarrow \left(\frac{u_w}{v_w}\right)^2 = \frac{P_w}{v_w}$$

$$\therefore u_w = \sqrt{P_w v_w} = \sqrt{Pv}$$

i.e. the sonic velocity is the maximum possible velocity.

$$\dot{m}_{max} = A\,\rho_w\,u_w = A\frac{\sqrt{P_w v_w}}{v_w} = A\sqrt{\frac{P_w}{v_w}} \ldots\ldots\ldots.* \sqrt{\frac{P_w}{v_w}}$$

$$\Rightarrow \dot{m}_{max} = A\sqrt{\frac{P_w^2}{P_w v_w}} = AP_w\sqrt{\frac{1}{P_w v_w}} = AP_w\sqrt{\frac{1}{P_1 v_1}} = AP_w\sqrt{\frac{1}{P_2 v_2}}$$

To find Pw, the following equation is used,

$$ln\left(\frac{P_1}{P_w}\right)^2 + 1 - \left(\frac{P_1}{P_w}\right)^2 + 8\ \phi\frac{L}{d} = 0$$

to get Pw at any given P1

[<u>its derivation is H.W.]</u>

<u>Example -8.1-</u>

Over a 30 m length of a 150 mm vacuum line carrying air at 295 K, the pressure falls from 0.4 kN/m^2 to 0.13 kN/m^2. If the relative roughness e/d is 0.003 what is the approximate flow rate? Take that μ_{air} at 295 K = 1.8 x 10^{-5} Pa.s

<u>Solution:</u>

$$G^2\,ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - p_1^2)}{2P_1 v_1} + 4\ \phi\frac{L}{d}G^2 = 0$$

It is required the velocity or G for calculating Re that used to estimate Φ from Figure (3.7)-vol.I. i.e. the solution is by **<u>trial and error technique.</u>**

1- Assume Φ = 0.004

$$v_1 = \frac{1}{\rho_1} = \frac{RT}{P_1 Mwt}$$
$$= \frac{8.314(Pa.m^3/mol.K)295\,K[(10^3 mol/kmol)]}{(0.4 * 10^3\ Pa)29\ kg/kmol}$$
$$= 211.434\,m^3/kg$$

$$G^2 ln\left(\frac{0.4}{0.13}\right) + \frac{(0.13 * 10^3 - 0.4 * 10^3)}{2(0.4 * 10^3)211.434} + 4\ (0.0004)\frac{30}{0.15}G^2 = 0$$

$$\Rightarrow 4.324\ G^2 = 0.846 \Rightarrow\Rightarrow G = 0.44\,kg/m^2.s$$

$$Re = G\frac{d}{\mu} = 3686 \Rightarrow\Rightarrow \phi = 0.005(figure\ 3.7)$$

2- Assume Φ = 0.005

$\Rightarrow$ 1.124 G^2 + 4 G^2 = 0.846 $\Rightarrow$ G = 0.41 $kg/m^2.s$

Re = G d / μ = 3435 $\Rightarrow$ Φ = 0.005 (Figure 3.7)

K.E. = $G^2 \ln(P_1/P_2)$ = (0.41)2 ln(0.4/0.13)

= 0.189 $kg^2/(m^4.s^2)$

Press.E. = $(P_2^2 - P_1^2) / (2 P_1 v_1)$

= – 0.846 $kg^2/(m^4.s^2)$

Frc.E. = 4 Φ L/d G^2 = 0.6724 $kg^2/(m^4.s^2)$

$[(P_1 - P_2) / P_1]$ % = 67.5%

Example -8.2-

A flow of 50 m^3/s methane, measured at 288 K and 101.3 kPa has to be delivered along a 0.6 m diameter line, 3km long a relative roughness e = 0.0001 m linking a compressor and a processing unit. The methane is to be discharged at the plant at 288 K and 170 kPa, and it leaves the compressor at 297 K. What pressure must be developed at the compressor in order to achieve this flow rate?

Take that μ_{CH4} at 293 K = 0.01 x 10^{-3} Pa.s

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - p_1^2)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$

$\Delta T/L = 11^0C/3000$ m = $0.00366^0C/m$ = 0.0366 $^0C/10$ m

= $0.366^0C/100m$ = $3.66^0C/1000$ m

$$G = \frac{\dot{m}}{A} = \frac{Q\rho}{A} = \frac{Q}{Av}$$

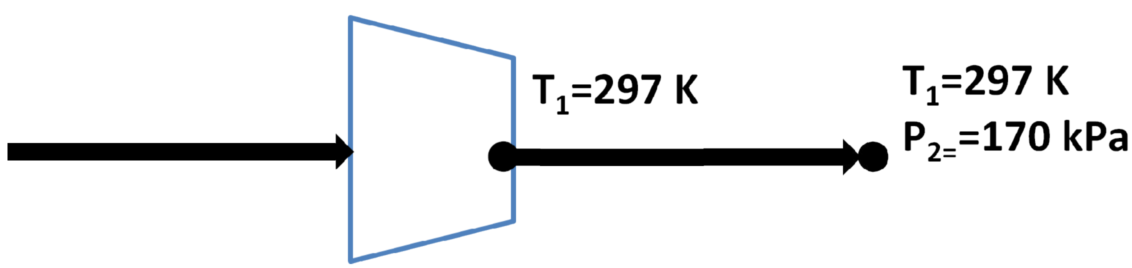
$$v = \frac{RT}{PMwt} = \frac{8.314(Pa.m^3/mol.K)288\,K[(10^3 mol/kmol)]}{(101.3 * 10^3 Pa)16\,kg/kmol}$$

$$= 1.477\ m^3/kg$$

$\Rightarrow$ G = (50) / [(π/4 0.6^2)(1.477)] = 119.7 $kg/m^2.s$

Since the difference in temperature is relatively small, therefore the processes could be consider isothermal at (T = Tm),

Tm = (297 + 288)/2 = 293 K



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$$P_1 v_1 = \frac{RT_m}{Mwt} = \frac{8.314(Pa.m^3/mol.K)293\ K}{16\,kg/kmol}$$
$$= 1.5225 * 10^5 Pa.m^3/kg \qquad or (J/kg \equiv m^2/s^2)$$

Re = G d / μ = 119.7(0.6)/0.01 x 10^{-3} = 7.182 x 10^6,

e/d = 0.0001 / 0.6 = 0.00016 ⇒ Φ = 0.0015 (Figure 3.7)

$$(119.7)^2\ ln\left(\frac{P_1}{170*10^3}\right)+\frac{(170*10^3-P_1^2)}{(16\,kg/kmol)}+4(0.0015)\frac{3000}{0.6}(119.7)^2=0$$

⇒ ln P_1 – 2.292 x 10^{-10} P_1^2 + 24.58 = 0 $\Rightarrow P_1 = \sqrt{\dfrac{ln P_1 + 24.58}{2.292*10^{-10}}}$

Solution by trial and error

P1 Assumed	**200 x 10^3**	**400.617 x 10^3**	**404.382 x 10^3**	**404.432 x 10^3**
P1 Calculated	**400.617 x 10^3**	**404.382 x 10^3**	**404.432 x 10^3**	**404.433 x 10^3**

⇒ P_1 = 404.433 x 10^3 Pa

K.E. = G^2 ln(P_1/P_2) = 12418 $kg^2/(m^4.s^2)$

Press.E. =– 442253 $kg^2/(m^4.s^2)$

Frc.E. = 429842 $kg^2/(m^4.s^2)$

[(P_1 – P_2) / P_1] % = 58.5%

Example -8.3-

Town gas, having a molecular weight 13 kg/kmol and a kinematic viscosity of 0.25 stoke is flowing through a pipe of 0.25 m I.D. and 5 km long at a rate of 0.4 m^3/s and is delivered at atmospheric pressure. Calculate the pressure required to maintain this rate of flow. The volume of occupied by 1 kmol and 101.3 kPa may be taken as 24 m3. What effect on the pressure required would result if the gas was delivered at a height of 150 m (i) above and (ii) below its point of entry into the pipe? e = 0.0005 m.

Solution:

P2 = P1 = 101.3 kPa

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{\left(P_2^2 - p_1^2\right)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$

$$G = \frac{\dot{m}}{A} = \frac{Q\rho}{A} = \frac{Q}{Av}$$

$$v = 24 \frac{m^3}{kmol}\left(\frac{1}{13\,kg/kmol}\right) = 1.846\, m^3/kg$$

⇒ G = (0.4) / [(π/4 0.25^2)(1.846)] = 4.414 kg/m^2.s
Re = G d / μ = G d / (ρ ν) = 4.414 (0.25) / [(1/1.846) 0.25 x 10^{-4}]
= 8.1489 x 10^4, e/d = 0.0005 / 0.25 = 0.002 ⇒ Φ = 0.0031 (Figure 3.7)

As first approximation the kinetic energy term will be omitted

$$\frac{-\Delta P}{v_m} = \frac{(P_1 - P_2)}{v_m} = 4\ \phi \frac{L}{d} G^2, \quad v_2 = 1.846\ 1.846\, m^3/kg,\ v_1 = \frac{RT}{P_1 . Mwt}$$

⇒ υ_1 = (8314) (289) / [(P1) 13] = 184.826 x 10^3/P1
υ_m = (υ_1 + υ_2) / 2 = [(184.826 x 10^3/P1) + 1.846]/2
= [92413.3+0.923 P1]/ P1

$$\Rightarrow \frac{(P_1 - 101.3 * 10^3) P_1}{92413.3 + 0.923 P_1} = 4(0.0031)\frac{5000}{0.25}(4.414)^2$$

$$\Rightarrow P_1^2 - 101.3 * 10^3 P_1 = 4831.9(92413.3 + 0.923 P_1)$$
$$= 4.4653 * 10^8 + 4459.8 P_1$$

P12 – 105.76 x 103 P1 – 4.4653 x 108 = 0
either P1 = 109.825 x 103 Pa
or P1 = – 4065.8 ------neglect
K.E. = G2 ln(P1/P2) = 1.5744 kg²/(m⁴.s²)
Press.E. =– 4831.9 kg²/(m⁴.s²)
Frc.E. = 4831.9 kg²/(m⁴.s²)
[(P1 – P2) / P1] % = 7.7 %

$$\boxed{x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}}$$

∴ The first approximation is justified

- If use the equation of the terms;

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - p_1^2)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$ ---- Neglect the kinetic energy term

$$\frac{(P_2^2 - p_1^2)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$

$$P_1 v_1 = \frac{RT}{Mwt} = \frac{8.314(Pa.m^3/kmol.K)}{13\,kg/kmol} = 184.8266 * 10^3 \left(J/kg \equiv m^2/s^2\right)$$

$$\Rightarrow p_1^2 = P_2^2 + \ 2P_1 v_1 \left(4\ \phi \frac{L}{d} G^2\right) = 0$$

= (101.3 x 103)^2 + 2(184.8266 x 10^3) [4(0.0031)(5000/0.25)(4.414)^2]
⇒P_1^2 = 1.20478 x 10^{10} ⇒P1 = 109.762 x 10^3 Pa

- If the pipe is not horizontal, the term (g dz) must be included in equation (**) or the term (g $\Delta z/\upsilon_m^2$) to integration of this equation [i.e. General equation of energy apply to compressible fluid in horizontal pipe with no shaft work]

υ_m = 1.7644 m^3/kg, υ_{air} = (8314 x 289)/(101.3 x 10^3 x 29) = 0.8179 m^3/kg
ρm = 0.5668 kg / m^3, ρair = 1.223 kg / m^3

- As gas is less dense than air, υm is replaced by (υ_{air} - υ_m) in potential energy term;

$$\frac{g\Delta z}{(v_{air}-v_m)^2}=\frac{9.81(150)}{(-0.9456)^2}=1642.55kg^2/m^4.s^2 \quad and \quad \frac{g\Delta z}{(v_{air}-v_m)}=1555Pa$$

(i) Point ❷ 150 m above point ❶ ⇒ P1 = 109.762 x 10^3 - 1555
= 108.207 x 103 Pa
(ii) Point ❷ 150 m below point ❶ ⇒ P1 = 109.762 x 10^3 - 1555
= 108.207 x 10^3 Pa ⇒ P1 = 109.762 x 103 + 1555 = 111.317 x 10^3 Pa

Example -8.4-

Nitrogen at 12 MPa pressure fed through 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 1.25 kg/s. What will be the drop in pressure over a 30 m length of pipe for isothermal flow of the gas at 298 K? e = 0.0005 m, μ = 0.02 mPa.s

Solution:

P1 = 12 MPa

First approximation [neglect the kinetic energy]

$$\frac{(P_2^2-p_1^2)}{2P_1v_1}+4\ \Phi\frac{L}{d}G^2=0$$

$$P_1v_1=\frac{RT}{Mwt}=\frac{8.314(Pa.m^3/kmol.K)\,298\,K}{28\,kg/kmol}=88484.7\left(J/kg\equiv m^2/s^2\right)$$

$$G=\frac{\dot{m}}{A}=\frac{1.25}{\pi/(4(0.025)^2)}=2546.48\ kg/m^2.s$$

Re = G d / μ = 2546.48 (0.025) / 0.02 x 10^{-3} = 3.183 x 10^6

, e/d = 0.0002 ⇒ Φ = 0.0017 (Figure 3.7)

P_2^2 = (12 x $10^6)^2$ **–2(88484.7) [4(0.0017)(30/0.025)(2546.48)2]**

⇒ P2 = 11.603 x 10^6 Pa

K.E. = G^2 ln(P1/P2) = 2.1816 x 10^5 $kg^2/(m^4.s^2)$

Press.E. = – 529.492 x 10^5 $kg^2/(m^4.s^2)$

Frc.E. = 529.14 x 10^5 $kg^2/(m^4.s^2)$

[(P1 – P2) / P1] % = 3.3 %

∴ the first approximation is justified

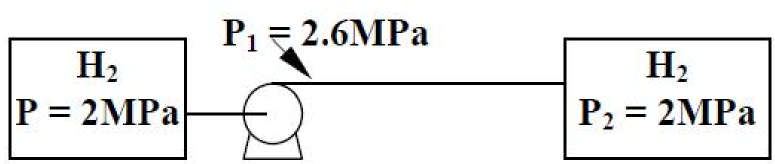
<u>Example -8.5-</u>

Hydrogen is pumped from a reservoir at 2 MPa pressure through a clean horizontal mild steel pipe 50 mm diameter and 500 m long. The downstream pressure is also 2 MPa. And the pressure of this gas is raised to 2.6 MPa by a pump at the upstream end of the pipe. The conditions of the flow are isothermal and the temperature of the gas is 293 K. What is the flow rate and what is the effective rate of working of the pump if η = 0.6 e = 0.05 mm, μ = 0.009 mPa.s.

<u>Solution:</u>

- First approximation [neglect the kinetic energy]

$$\frac{(P_2^2 \quad p_1^2)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$



$$P_1 v_1 = \frac{RT}{Mwt} = \frac{8.314 (Pa.m^3/kmol.K)\, 293\, K}{2\, kg/kmol} = 121.8 * 10^4\ (J/kg \equiv m^2/s^2)$$

P1 = 2.6 MPa, P2 = 2 MPa, $-\Delta P_f$ = P1 – P2 = 0.6 x 10^6 Pa

$\rho_m = 1/\upsilon_m = P_m$ Mwt/RT = $(2.3 \times 10^6)^{\ 2}$ / (8314 x 293) = 1.89 kg/m^3

$\Phi Re^2=(-\Delta Pf/L)(\rho_m d^3/4\mu^2)$

$=[(0.6x10^6)/(500)][(1.89)(0.05)^3/(4)(0.009x10^{-3})^2]= 8.75x10^8$

$e/d = 0.001 \Rightarrow$ Figure (3.8) $Re = 5.9 \times 10^5 \Rightarrow G = 5.9 \times 10^5 (0.009x10^{-3}) / (0.05)$

$\Rightarrow G = 106.2$ kg / m^2.s

$Re = 5.9 \times 10^5$, $e/d = 0.001 \Rightarrow \Phi = 0.0025$ (Figure 3.7)

$$G^2 = \frac{(P_2^2 - p_1^2)}{2P_1 v_1(-4\phi L/d)} = \frac{(2 \times 10^6)^2 (2.6 \times 10^6)^2}{(2 * 121.8 * \times 10^4)[-4(0.0025)(500/0.05)]} = 11330$$

$\Rightarrow$ G = 106.44 kg / m2.s ------------------------∴ ok

K.E. = G^2 ln(P1/P2) = 2.9726 x 10^3 kg^2/(m^4.s^2)

Press.E. = – 1133.005 x 10^3 kg^2/(m^4.s^2)

Frc.E. = 1132.94736 x 10^3 kg^2/(m^4.s^2)

[(P1 – P2) / P1] % = 3.3 %

$$power = \frac{\dot{m} P_1 v_1 ln(P_1/P_2)}{\eta} = \frac{0.209(121.8 * 10^4) ln(2.6/2)}{0.6} = 111.3\ kW$$

<u>Example -8.6-</u>

In the synthetic ammonia plant the hydrogen is fed through a 50 mm diameter steel pipe to the converters. The pressure drop over the 30 m length of pipe is 500 kPa, the pressure at the downstream end being 7.5 MPa. What power is required in order to overcome friction losses in the pipe? Assume isothermal expansion of the gas at 298 K. What error introduced by assuming the gas to be an incompressible fluid of density equal to that at the mean pressure in the pipe? μ = 0.02 mPa.s.

Solution:

P_2=7.5 MPa, $P_1 = P_2 + (-\Delta Pf)$ = 7.5 MPa + 0.5 MPa = 8MPa = 8 x 10^6 Pa

The pressure (P_m) = (P_1 + P_2)

[(P1 – P2) / P1] % = 6.25 %

$\rho_m=P_m.Mwt/RT=7.75*10^6(2)/(4314*298)$

$=6.256$ kg/ m^3

For incompressible fluids

$$\frac{-\Delta P}{\rho_m} = 4\ \phi \frac{L}{d} u^2$$

$$\Rightarrow -\Delta P\, \rho_m - 4\ \phi \frac{L}{d} u^2 \rho_m^2 - 4\ \phi \frac{L}{d} G^2 \ \Rightarrow G^2 - \frac{-\Delta P\, \rho_m}{4\ \phi\, L/d}$$

Assume Φ = 0.003

$\Rightarrow G^2$ = 434,444.444 $kg^2/m^4.s^2 \Rightarrow$ G = 659.124 kg/m2.s

$\Rightarrow$ Re =1.647 x 10^6, and Φ = 0.003 $\Rightarrow$ from Figure (3.7)

e/d = 0.00189

$\Rightarrow$ e = 0.09 mm (this value is reasonable for steel)

For compressible fluids

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - p_1^2)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$

$$G^2 \ln\left(\frac{8}{7.5}\right) + \frac{(7.5*10^6)^2 - (8*10^6)^2}{2[8314(298)/2]} + 4\,(0.003)\frac{30}{0.05} G^2 = 0$$

$\Rightarrow G^2$ = 430,593.418 $kg^2/m^4.s^2 \Rightarrow$ G = 656.2 $kg/m^2.s$

Very little error is made by the simplifying assumption in this particular case.

$$power = \frac{\dot{m}\, P_1 v_1 ln(P_1/P_2)}{\eta}$$

$$= \frac{\left(656.2 * \frac{\pi}{4} * (0.005)^2\right)(123.8786 * 10^6) ln\, 0.209(121.8 * 10^4) ln(8/7.5)}{0.6}$$

$$= 171.7 \;\; kW$$

<u>Example -8.7-</u>

A vacuum distillation plant operating at 7 kPa pressure at top has a boil-up rate of 0.125 kg/s of xylene. Calculate the pressure drop along a 150 mm bore vapor pipe used to connect the column to the condenser. And also calculate the maximum flow rate if L = 6 m, e = 0.0003 m, Mwt = 106 kg/kmol, T = 338 K, μ = 0.01 mPa.s.

<u>Solution:</u>

$$G^2 ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 \quad p_1^2)}{2P_1 v_1} + 4 \;\; \Phi \frac{L}{d} G^2 = 0$$

$G = 0.125 / [\pi/4\ (0.15)^2] = 7.074\ kg/m^2.s$

P_1 = 7 kPa, P_2 = Pressure at condenser

$$P_1 v_1 = \frac{RT}{Mwt} = \frac{8.314(Pa.m^3/kmol.K)\, 338\, K}{106\, kg/kmol}$$

$$= 26510.68\, (J/kg \equiv m^2/s^2)$$

$Re = G\, d / \mu = 7.074(0.15)/0.01 \times 10^{-3} = 1.06 \times 10^5$, $e/d = 0.002$

$\Rightarrow \Phi = 0.003$ (Figure 3.7)

$$ln\left(\frac{7 * 10^3}{P_2}\right) + 3.769 * 10^{-7}[P_2 - (7 * 10^3)^2] + 4\,(0.003)(6/0.15) = 0$$

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$$\Rightarrow P_2^2 = (7 * 10^3)^2 - \frac{\ln(7 * 10^3/P_2) + 0.48}{3.769 * 10^{-7}}$$

$$\Rightarrow P_2 = \sqrt{(7 * 10^3)^2 - \frac{\ln(7 * 10^3/P_2) + 0.48}{3.769 * 10^{-7}}}$$

Solution by trial and error

P_2 Assu.	5×10^3	6.8435×10^3	6.904×10^3	6.9057×10^3
P_2 Calc.	6.8435×10^3	6.904×10^3	6.9057×10^3	6.9058×10^3

$-\Delta P = P1 - P2 = (7 - 6.9058) \times 10^3 = 94.2\text{Pa}$

[(P1 – P2) / P1] % = 0.665 % we can neglect the K.E. term in this problem

<u>H.W. resolve this example with neglecting the K.E. term</u>

For maximum flow rate calculations

$$\dot{m}_{max} = A\, P_w \sqrt{1/P_1 v_1} \Rightarrow G_{max} = P_w \sqrt{1/P_1 v_1}$$

To estimate Pw

$$\ln\left(\frac{P_1}{P_w}\right)^2 + 1 - \left(\frac{P_1}{P_w}\right)^2 + 8\,\Phi\,\frac{L}{d} = 0$$

Let $X \equiv (P1/Pw)^2$

$\Rightarrow \ln(X) + 1 - X + 8\,\Phi\, L/d = 0 \Rightarrow X = 1.96 + \ln(X)$

Solution by trial and error

X Assu.	1.2	2.14	2.72	2.96	3.074	3.086	3.087
X Calc.	2.14	2.72	2.96	3.074	3.086	3.087	3.087

$\Rightarrow X = 3.087 = (P1/Pw)^2 \Rightarrow Pw = P1/(3.087)^{0.5} = 3984$ Pa

$\therefore$ the system does not reach maximum velocity (<u>H.W. explain)</u>

$\Rightarrow G_{max} = 3984 / (26510.68)^{0.5} = 24.47\ \text{kg/m}^2.\text{s}$

Example -8.8-

A vacuum system is required to handle 10 g/s of vapor (molecular weight 56 kg/kmol) so as to maintain a pressure of 1.5 kN/m^2 in a vessel situated 30 m from the vacuum pump. If the pump is able to maintain a pressure of 0.15 kN/m^2 at its suction point, what diameter of pipe is required? The temperature is 290 K, and isothermal conditions may be assumed in the pipe, whose surface can be taken as smooth. The ideal gas law is followed. Gas viscosity μ = 0.01 mN s/m^2.

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - p_1^2)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$

$$G = \frac{\dot{m}}{\pi/4d^2} = \frac{10 \times 10^{-3}}{\pi/4d^2} = 0.0127d^2$$

Re = G d / μ = 1273.25 d^{-1} --------------------------------(1)

$$P_1 v_1 = \frac{RT}{Mwt} = \frac{8.314(Pa.m^3/kmol.K)\ 290\ K}{50\ kg/kmol}$$
$$= 43054.64(J/kg \equiv m^2/s^2)$$

$$d = \left[\frac{52.97 - 0.019\, d^{-3}\,\phi}{3.733 \times 10^{-4}}\right]^{-1/4} \quad \dots\dots\dots\dots\dots\dots\dots (2)$$

<u>Assume smooth pipe</u>

Solution by trial and error

	Eq.(1)	Figure (3.7)	Eq.(2)
Assume d = 0.1	⇒ Re = 1.3 x 10^{-4}	⇒ Φ = 0.0038	⇒ d = 0.0515
d = 0.0515	⇒ Re = 2.5 x 10^{-4}	⇒ Φ = 0.0028	⇒ d = 0.0516

∴ d = 0.0516 m.

8.3.2 Adiabatic Flow of an Ideal Gas in a Horizontal Pipe

The general energy equation of a steady-state flow system is: -

dH + g dz + u du = dq -dWs

For adiabatic conditions (dq = 0) and in horizontal pipe (dz = 0) with no shaft work dWs = 0)

dH=dU+d(Pv)
$c_p dT = c_v dT + R dT$
$c_p = c_v + R$

⇒ dH + u du = 0

$$but\ G = \frac{\dot{m}}{A} = \rho u \Rightarrow u = vG$$

⇒ dH + G^2 υ d υ = 0

we have dH = cp dT, and dPυ = RdT

⇒dT = dPυ/R = dPυ/(cp – cv)

⇒ dH = cp [dPυ/(cp – cv)] = (cp / cv) /[(cp – cv)/ cv] dPυ

= [γ/(γ – 1)] dPυ

$$\therefore \frac{\gamma}{\gamma - 1} dPv + G^2 v dv = 0$$ The integration of this equation gives:-

$$\therefore \frac{\gamma}{\gamma-1} P_1 v_1 + \frac{G^2}{2} v_1^2 = \frac{\gamma}{\gamma-1} P_2 v_2 + \frac{G^2}{2} v_2^2 = \frac{\gamma}{\gamma-1} P\, v + \frac{G^2}{2} v^2 = K$$

<u>This equation is used to estimate the downstream pressure P2</u>

To estimate the downstream specific volume υ2 the procedure is as follow

$$\frac{\gamma}{\gamma-1} P\, v = K - \frac{G^2}{2} v^2 \Rightarrow P = \left(\frac{\gamma}{\gamma-1}\right)\left[\frac{K}{v} - \frac{G^2}{2} v^2\right]$$

$$\Rightarrow dP = \left(\frac{\gamma}{\gamma-1}\right)\left[-\frac{K}{v^2} - \frac{G^2}{2}\right] dv \qquad \div v$$

$$\Rightarrow \frac{dP}{v} = \left(\frac{\gamma}{\gamma-1}\right)\left[-\frac{K}{v^3} - \frac{G^2}{2v}\right] dv$$

$$\Rightarrow \int_{P_1}^{P_2} \frac{dP}{v} = \left(\frac{\gamma}{\gamma-1}\right)\left[\frac{K}{2}\left(\frac{1}{v_2^2}-\frac{1}{v_1^2}\right)-\frac{G^2}{2}\,ln\,\left(\frac{v_2}{v_1}\right)\right]$$

$$but\,,K = \frac{G^2}{2}v_1^2 + \frac{\gamma}{\gamma-1}P_1 v_1$$

$$\Rightarrow \int_{P_1}^{P_2} \frac{dP}{v} = \left(\frac{\gamma}{\gamma-1}\right)\left[\frac{G^2}{4}v_1^2\left(\frac{1}{v_2^2}-\frac{1}{v_1^2}\right)+\frac{\gamma}{\gamma-1}\frac{P_1 v_1}{2}\left(\frac{1}{v_2^2}-\frac{1}{v_1^2}\right)-\frac{G^2}{2}ln\,\left(\frac{v_2}{v_1}\right)\right]$$

$$= \frac{\gamma-1}{4\gamma}G^2\left[\left(\frac{v_1}{v_2}\right)^2-1-2ln\left(\frac{v_2}{v_1}\right)\right]+\frac{P_1 v_1}{2}\left(\frac{1}{v_2^2}-\frac{1}{v_1^2}\right)$$

$$but\,,\ G^2\,ln\left(\frac{v_2}{v_1}\right)+\int_{P_1}^{P_2}\frac{dP}{v}+4\phi\frac{L}{d}\ G^2=0$$

The general equation of energy apply to compresible fluid in horizontal pipe with no shaft work

$$G^2\,ln\left(\frac{v_2}{v_1}\right)+\frac{\gamma-1}{4\gamma}G^2\left[\left(\frac{v_1}{v_2}\right)^2-1-2ln\left(\frac{v_2}{v_1}\right)\right]+\frac{P_1 v_1}{2}\left(\frac{1}{v_2^2}-\frac{1}{v_1^2}\right)+4\phi\frac{L}{d}\ G^2=0 \qquad \cdots\cdots \times \frac{2}{G^2}$$

$$\Rightarrow 2\,ln\left(\frac{v_2}{v_1}\right)-\frac{\gamma-1}{\gamma}ln\left(\frac{v_2}{v_1}\right)+\frac{\gamma-1}{2\gamma}\left|\left(\frac{v_1}{v_2}\right)^2-1\right|+\frac{P_1}{v_1\ G^2}\left|\left(\frac{v_1}{v_2}\right)^2-1\right|+8\phi\frac{L}{d}=0$$

$$\Rightarrow \frac{2\gamma-\gamma+1}{\gamma}\ ln\left(\frac{v_2}{v_1}\right)+\left[\left(\frac{v_1}{v_2}\right)^2-1\right]\left[\frac{\gamma-1}{2\gamma}+\frac{P_1}{v_1\ G^2}\right]+8\phi\frac{L}{d}=0$$

$$\boxed{\Rightarrow \frac{\gamma+1}{\gamma}\ ln\left(\frac{v_2}{v_1}\right)+\left[\frac{\gamma-1}{2\gamma}+\frac{P_1}{v_1\ G^2}\right]\left[\left(\frac{v_1}{v_2}\right)^2-1\right]+8\phi\frac{L}{d}=0}$$

This equation is used to estimate the downstream specific volume υ_2

8.3.2.1 Maximum Velocity in Adiabatic Flow

For constant upstream conditions, the maximum flow through the pipe is found by differentiating (G) with respect to (υ_2) of the last equation and putting ($dG/d\upsilon_2$) equal to zero.

The maximum flow is thus shown to occur when the velocity at downstream end of the pipe is the sonic velocity.

$$i.e.\ \frac{dG}{d\upsilon_2} = 0 \Rightarrow u_w = \sqrt{\gamma P_2 v_2} \Rightarrow G_{max} = \frac{\sqrt{\gamma P_2 v_2}}{v_2} = \sqrt{\frac{\gamma P_2}{v_2}}$$

Note: -

In isentropic (or adiabatic) flow [$P_1\upsilon_1 \neq P_2\upsilon_2$] where, in these conditions [[$P_1\upsilon_1 \neq P_2\upsilon_2$] γ]i.e. $u_w = \sqrt{\gamma P_2 v_2} \neq \sqrt{\gamma P_1 v_1}$

Typical values of (γ) for ordinary temperatures and pressures are: -

i- For monatomic gases such as He, Ar (γ = 1.67)

ii- For diatomic gases such as H_2, N_2, CO (γ = 1.4)

iii- For tritomic gases such as CO_2 (γ = 1.3)

Example -8.9-

Air, at a pressure of 10 MN/m^2 and a temperature of 290 K, flows from a reservoir through a mild steel pipe of 10 mm diameter and 30 m long into a second reservoir at a pressure P_2. Plot the mass rate of flow of the air as a function of the pressure P_2. Neglect any effects attributable to differences in level and assume an adiabatic expansion of the air. μ = 0.018 mN s/m^2, γ = 1.36.

Solution:

$$\boxed{\Rightarrow \frac{\gamma+1}{\gamma}\ ln\left(\frac{v_2}{v_1}\right) + \left[\frac{\gamma-1}{2\gamma} + \frac{P_1}{v_1\ G^2}\right]\left[\left(\frac{v_1}{v_2}\right)^2 - 1\right] + 8\phi\frac{L}{d} = 0}$$

$$v_1 = \frac{RT}{P_1 Mwt} = \frac{8.314\,(Pa.m^3/kmol.K)\;290\;K}{10 * 10^6\;Pa(29\,kg/kmol)} = 8.314 * 10^{-3}\;\;m^3/kg$$

$$\Rightarrow 1.735\,ln\left(\frac{v_2}{8.314 * 10^{-3}}\right) + \left[0.132 + \frac{1.2028 * 10^9}{G^2}\right]\left[\left(\frac{8.314 * 10^{-3}}{v_2}\right)^2 - 1\right] + 24000\phi = 0$$

$$\Rightarrow \left(\frac{8.314 * 10^{-3}}{v_2}\right)^2 = 1 - \frac{1.735\,ln\left(\frac{v_2}{8.314 * 10^{-3}}\right) + 24000\phi}{0.132 + \frac{1.2028 * 10^9}{G^2}}$$

$$\Rightarrow v_2 = \frac{8.314 * 10^{-3}}{\sqrt{1 - \frac{1.735\,ln\left(\frac{v_2}{8.314 * 10^{-3}}\right) + 24000\phi}{0.132 + \frac{1.2028 * 10^9}{G^2}}}} ------(1)$$

$$Re = \frac{Gd}{\mu} = 555.6\,G ---------------(2)$$

$$\frac{\gamma+1}{\gamma}P_1 v_1 + G^2 v_1^2 = \frac{\gamma}{\gamma-1}P_2 v_2 + \frac{G^2}{2}v_2^2 \qquad \Rightarrow \frac{\gamma}{\gamma-1}P_2 v_2 = \frac{\gamma}{\gamma-1}P_1 v_1 + \frac{G^2}{2}\left(v_1^2 - v_2^2\right)$$

$$\Rightarrow P_2 = \frac{P_1 v_1}{v_2} + \frac{\gamma-1}{2\gamma}\,\frac{G^2}{2}\left(v_1^2 - v_2^2\right)$$

$$\Rightarrow P_2 = \frac{83140}{v_2} + 0.132\;\frac{G^2}{2}\left(6.91 * 10^{-5} - v_2^2\right) -----(3)$$

1- at $P_2 = P_1 \Rightarrow G = 0$

eq.(2) Figure (3.7)

2- assume G = 2000 kg/m^2.s $\Rightarrow$ Re = 1.11 x $10^6 \Rightarrow \Phi = 0.0028$

Solution by trial and error

υ_2 Assumed	10 x 10^{-3}	9.44 x 10^{-3}
υ_2 Calculated eq.(1)	9.44 x 10^{-3}	9.44 x 10^{-3}

3- assume G = 3000 kg/m^2·s $\Rightarrow$ Re = 1.6 x $10^6 \Rightarrow \Phi = 0.0028$

Solution by trial and error

υ_2 Assumed	10 x 10^{-3}	11.8 x 10^{-3}
υ_2 Calculated eq.(1)	11.8 x 10^{-3}	11.81 x 10^{-3}

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G ($kg/m^2 s$)	υ_2 (m^3/kg)	P_2 (Mpa)
0	$8.314 * 10^{-3}$	10
2000	$9.44 * 10^{-3}$	8.8
3000	$11.81 * 10^{-3}$	7.013
3500	$16.5 * 10^{-3}$	5.01
4000	$25 * 10^{-3}$	3.37
4238	$39 * 10^{-3}$	2.04

<u>Example -8.10-</u>

Nitrogen at 12 MN/m^2 pressure is fed through a 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 0.4 kg/s. What will be the drop in pressure over a 30 m length of pipe assuming isothermal expansion of the gas at 300 K? What is the average quantity of heat per unit area of pipe surface that must pass through the walls in order to maintain isothermal conditions? What would be the pressure drop in the pipe if it were perfectly lagged? What would be the maximum flow rate in each case? Or what would be the Mach number? $\mu = 0.02\ mNs/m^2$, $\gamma = 1.36$, $e/d = 0.002$.

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - p_1^2)}{2P_1 v_1} + 4\ \phi \frac{L}{d} G^2 = 0$$

$$P_1 v_1 = \frac{RT}{Mwt} = \frac{8.314 (Pa.m^3/kmol.K)\ 300\ K}{28\ kg/kmol}$$

$$- 89078.6 (J/kg \equiv m^2/s^2)$$

$$G = \frac{m^{\cdot}}{\pi/4d^2} = 0.4/[\pi/4(0.25)^2] = 814.9\ kg/m^2.s \quad Re = \frac{Gd}{\mu} = 1.02 * 10^6$$

$e/d = 0.002 \Rightarrow \Phi = 0.0028$ Figure (3.7)

● Neglect the K.E. term

$\Rightarrow P_2^2 = P_1^2 - 2\, P_1\upsilon_1(4\ \Phi\ (L/d)\ G^2) = 1.4241 \times 10^{14}$

$\Rightarrow P2 = 11.93 \times 10^6$ Pa

K.E. = $G^2 \ln(P_1/P_2)$ = 3.885×10^4 $kg^2/(m^4.s^2)$

Press.E. = – 940.24 x 104 $kg^2/(m^4.s^2)$

Frc.E. = 892.5 x 104 $kg^2/(m^4.s^2)$

∴ the neglecting of Kinetic energy term is OK

$[(P_1 - P_2) / P_1]$ % = 0.583 %

$\Rightarrow -\Delta P = P_1 - P_2 = 0.07 \times 10^6$ Pa

isothermal horizontal no shaft work

$$\cancel{dH} + g\,\cancel{dz} + u\,du = dq - \cancel{dWs}$$

$\Rightarrow u\,du = dq \Rightarrow q = \Delta u^2/2 = u_1^2/2$ [since the veloity in the plant is taken as zero]

$\Rightarrow q = (G\,\upsilon_1)^2 /2 = [814.9(89078.6/12 \times 10^6)]^2 / 2 = 18.3$ J/kg

The total heat pass through the wall **= 0.4 (18.3) = 7.32 W**

Heat flux **q''=** **$= q_T / (\pi\ d\ L) = 7.32 / [\pi\ (0.025)\ 30] = 3.1\ W/m^2$**

It is clear that the heat flux is very low value that could be considered the process is adiabatic.

For adiabatic conditions

For adiabatic conditions

$$\frac{\gamma+1}{\gamma}\, ln\left(\frac{v_2}{v_1}\right)+\left[\frac{\gamma-1}{2\gamma}+\frac{P_1}{v_1\ G^2}\right]\left[\left(\frac{v_1}{v_2}\right)^2-1\right]+8\phi\frac{L}{d}=0$$

$$\Rightarrow v_2=\frac{v_1}{\sqrt{1-\dfrac{\frac{\gamma+1}{\gamma}\, ln\left(\frac{v_2}{v_1}\right)+8\phi\frac{L}{d}}{\frac{\gamma-1}{2\gamma}+\frac{P_1}{v_1\ G^2}}}}\qquad \Rightarrow v_2=\frac{7.423*10^{-3}}{\sqrt{1-\dfrac{1.714\, ln\left(\frac{v_2}{7.423*10^{-3}}\right)+26.88}{0.143+2434.336}}}$$

Solution by trial and error

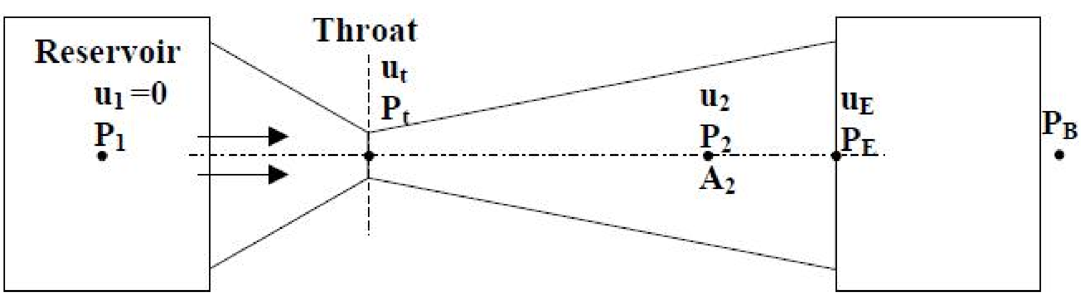
υ_2 Assumed	10×10^{-3}	7.5×10^{-3}	$\Rightarrow$ **$\upsilon_2 = 7.46 \times 10^{-3}$ m³/kg**
υ_2 Calculated	7.5×10^{-3}	7.46×10^{-3}	

$$\Rightarrow P_2=\frac{P_1 v_1}{v_2}+\frac{\gamma-1}{2\gamma}\,\frac{G^2}{2}\left(v_1^2-v_2^2\right)\Rightarrow P_2=11.94*10^6 Pa$$

This value of P_2 in adiabatic conditions is very close to the value in isothermal condition since the actual heat flux is very small.

8.4 Converging-Diverging Nozzles for Gas Flow

Converging-diverging nozzles, sometimes known as "Laval nozzles", are used for expansion of gases where the pressure drop is large.



P_1: the pressure in the reservoir or initial pressure.

P_2: the pressure at any point in diverging section of the nozzle.

P_E: the pressure at exit of the nozzle.

P_B: the back pressure or the pressure at end.

$P_{critical}$: the pressure at which the velocity of the gas is sonic velocity.

Because the flow rate is large for high-pressure differentials, there is little time for heat transfer to take place between the gas and surroundings and the expansion is effectively isentropic [adiabatic + reversible]. In these conditions,

$$\frac{v_2}{v_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \Rightarrow v_2 = v_1 \left(\frac{P_2}{P_1}\right)^{-\frac{1}{\gamma}}$$

$$\frac{\Delta u^2}{2} + g\,\Delta z + \int_{P_1}^{P_2} v\,dp + W_s + F = 0$$

the genral energy equation for any type of fluid.

for gas flow from reservoir ($u_1 = 0$) at pressure (P_1) in a horizontal direction, with no shaft work, and by assuming F=0 this equation becomes

$$\frac{u_2^2}{2} \int_{P_1}^{P_2} v\,dp = 0$$

and the pressure energy term is,

$$\int_{P_1}^{P_2} v\,dp = v_1 P_1^{\frac{1}{\gamma}} \int_{P_1}^{P_2} P^{\frac{1}{\gamma}}\,dp = v_1 P_1^{\frac{1}{\gamma}} \left[\frac{P^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}\right]_{P_1}^{P_2} = v_1 P_1^{\frac{1}{\gamma}} \left(\frac{\gamma}{1-\gamma}\right)\left[P_2^{\frac{\gamma-1}{\gamma}} - P_1^{\frac{\gamma-1}{\gamma}}\right] - - \times \frac{P_1^{\frac{\gamma-1}{\gamma}}}{P_1^{\frac{\gamma-1}{\gamma}}}$$

$$\Rightarrow \int_{P_1}^{P_2} v\,dp = \left(\frac{\gamma}{\gamma-1}\right) P_1 v_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\Rightarrow u_2^2 = -2\int_{P_1}^{P_2} v\,dp = \left(\frac{2\gamma}{\gamma-1}\right) P_1 v_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

To estimate the velocity at any point downstream

we have, $G_2 = \dfrac{\dot{m}}{A_2} = \dfrac{u_2}{v_2} \Rightarrow A_2 = \dot{m}\dfrac{v_2}{u_2}$

Cross-sectional area at any point downstream

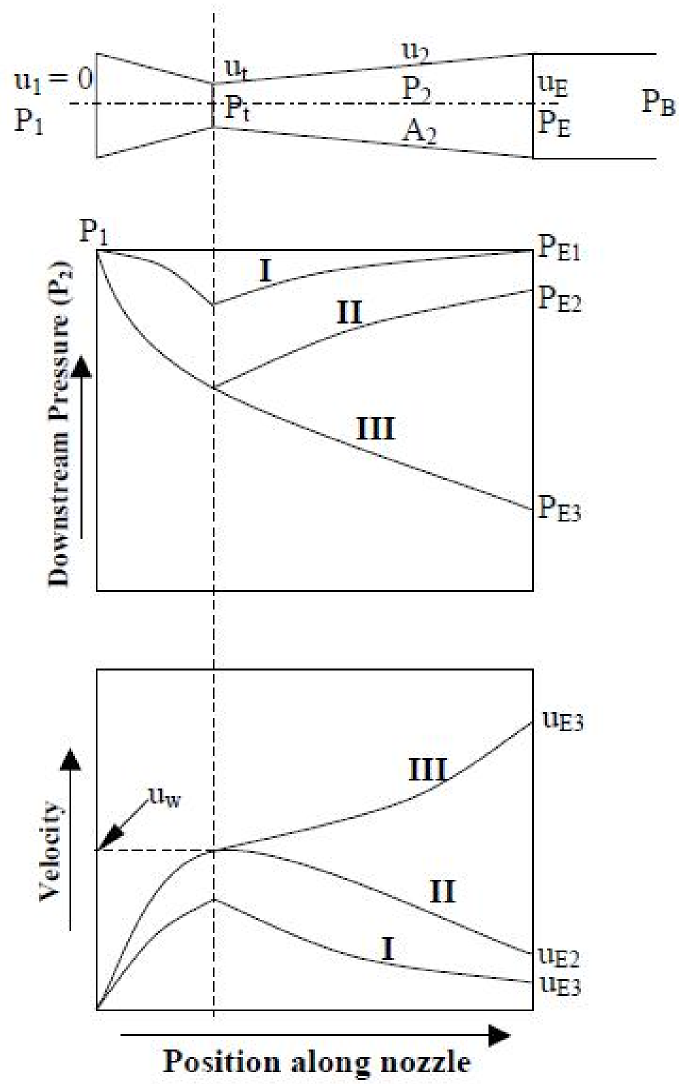
8.4.1 Maximum Velocity and Critical Pressure Ratio

Critical pressure is the pressure at which the gas reaches sonic velocity [i.e. Ma =1.0].

In converging-diverging nozzles, if the pressure ratio (P_2/P_1) is less than the critical pressure ratio ($P_{critical}/P_1$) (usually, ≈ 0.5) and the velocity at throat is then equal to the velocity of sound, the effective area for flow presented by nozzle must therefore pass through a minimum. Thus in a converging section the velocity of the gas stream will never exceed the sonic velocity, though supersonic velocities may be obtained in the diverging section of the converging-diverging nozzle.

Case (I) [PB high, Pt > Pcritical]

The pressure falls to a minimum at throat [lager than critical pressure] and then rises to a value ($P_{E1}=P_B$). The velocity increase to the maximum at throat [less than sonic velocity] and then decreases to a value of (u_{E1}) at the exit of the nozzle. [**Case (I)** is corresponding to conditions in a venturi meter operating entirely at subsonic velocities]



Case (II) [PB reduced, PB > (Pt = Pcritical)]

The pressure falls to a critical value at throat where the velocity is sonic. The pressure then rises to a value (P_{E2}=P_B) at the exit of the nozzle. The velocity rises to the sonic value at the throat and then falls to a value of (u_{E2}) at the exit of the nozzle.

Case (III) [PB low, PB < (Pt = Pcritical)]

The pressure falls to a critical value at throat and continues to fall to give an exit pressure (P_{E3}=P_B). The velocity rises to the sonic value at the throat and continues to increase to supersonic in the diverging section cone to a value (u_{E3}) at the exit of the nozzle.

With converging-diverging nozzle, the velocity increases beyond the sonic velocity [i.e. reach supersonic velocity] only if the velocity at the throat is sonic [i.e. critical pressure at throat] and the pressure at outlet is lower than the throat pressure.

8.4.2 The Pressure and Area for Flow

In converging-diverging nozzles, the area required at any point depend upon the ratio of the downstream to upstream pressure (P2/P1), and **it is helpful to establish the minimum value of (At = A2).**

$$A_2 = A_2 = \dot{m}\frac{v_2}{u_2} \rightarrow A_2^2 = \dot{m}^2\left(\frac{v_2}{u_2}\right)^2$$

$$but\ v_2 = v_1\left(\frac{P_2}{P_1}\right)^{-\frac{1}{\gamma}} \quad and\ u_2^2 = \left(\frac{2\gamma}{\gamma-1}\right)P_1 v_1\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\Rightarrow A_2^2 = \dot{m}^2\left(\frac{\gamma-1}{2\gamma}\right)\left[\frac{v_1^2(P_2/P_1)^{-\frac{2}{\gamma}}}{P_1v_1\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right] \qquad \Rightarrow A_2^2 = \left(\frac{\dot{m}^2 v_1(\gamma-1)}{2\gamma P_1}\right)\left[\frac{v_1^2(r)^{-\frac{2}{\gamma}}}{P_1v_1\left[1-(r)^{\frac{\gamma-1}{\gamma}}\right]}\right]; r = \frac{P_2}{P_1}$$

In the flow stream P_1 falls to P_2 at which minimum A_2 which could be obtain by;

$$\left(\frac{dA_2^2}{dr}\right)_{r=rc} = 0$$

$$\left(\frac{dA_2^2}{dr}\right) = 0 \Longrightarrow \left(\frac{\dot{m}^2 v_1(\gamma-1)}{2\gamma P_1}\right)\left[\frac{\left(1-r_c^{\frac{\gamma-1}{\gamma}}\right)\left(\frac{-2}{\gamma}\right)\left(r_c^{-\left(\frac{2+\gamma}{\gamma}\right)}\right)-\left(r_c^{\left(\frac{-2}{\gamma}\right)}\right)\left(-\frac{\gamma-1}{\gamma}r_c^{-\frac{1}{\gamma}}\right)}{\left\{1-(r)^{\frac{\gamma-1}{\gamma}}\right\}^2}\right] = 0$$

$$\Rightarrow \left(1-r_c^{\frac{\gamma-1}{\gamma}}\right)\left(\frac{-2}{\gamma}\right)\left(r_c^{\frac{-2-\gamma}{\gamma}}\right)+\left(r_c^{\left(\frac{-2}{\gamma}\right)}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(r_c^{-\frac{1}{\gamma}}\right) = 0 \Rightarrow \left(\frac{-2}{\gamma}\right)\left(r_c^{\frac{-2-\gamma}{\gamma}}\right)+\left(\frac{\gamma+1}{\gamma}\right)\left(r_c^{\frac{-3}{\gamma}}\right) = 0$$

$$\Rightarrow r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}; \gamma = \frac{c_p}{c_v}; r_c = \frac{P_{critical}}{P_1} \quad if\ \gamma = 1.4 \Rightarrow r_c = 0.528$$

$$\Rightarrow A_2^2 = \dot{m}^2\frac{(\gamma-1)}{2\gamma}\left(\frac{v_1}{P_1}\right)\left[\frac{\left(\frac{P_2}{P_1}\right)^{-\frac{2}{\gamma}}}{\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right]$$

The area at any Point downstream

$$and\ \dot{m}^2 = A_2^2\frac{2\gamma}{(\gamma-1)}\left(\frac{P_1}{v_1}\right)\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

The mass flow rate

$$and\ G_2^2 = \frac{\dot{m}^2}{A_2^2}\frac{2\gamma}{(\gamma-1)}\left(\frac{P_1}{v_1}\right)\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

The mass velocity

To find the maximum value of (G_2) i.e. $(G_2)_{max}$, set $(dG_2^2/dr = 0)$ where, $r = P_2/P_1$ to get the following equation

Example -8.11-

Air enters at a pressure of 3.5 MPa and a temperature of 500^{0}C. The air flow rate th rough the nozzle is 1.3 kg/s and it leaves the nozzle at a pressure of 0.7 MPa. The expansion of air may be considered adiabatic. Calculate the area of throat and the exit area. Take γ = 1.4.

Solution:

$$A_2^2 = \dot{m}^2 \frac{(\gamma-1)}{2\gamma}\left(\frac{v_1}{P_1}\right)\left[\frac{\left(\frac{P_2}{P_1}\right)^{-\frac{2}{\gamma}}}{\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right]$$

$$v_1 = \frac{RT_1}{P_1 Mwt} = \frac{8.314\,(Pa.m^3/kmol.K)\;773.15\;K}{3.5*10^6\;Pa(29\;kg/kmol)} = 0.0633\;m^3/kg$$

r =P_2/P_1, r_c =$P_{critical}/P_1$ = $P_{critical}/P_1$ $\Rightarrow r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528$

$\Rightarrow P_{critical} = P_t$ = 0.528 (3.5 MPa) = 1.85 MPa
but P_2 = 0.7 MPa [i.e. $P_2 < P_t$] $\Rightarrow$ The case is **(III)**

at throat

$$A_t^2 = (1.3)^2\frac{0.4}{2.8}\left(\frac{0.0633}{3.5*10^6}\right)\left[\frac{(0.528)^{-\frac{2}{1.4}}}{\left[1-(0.528)^{\frac{0.4}{1.4}}\right]}\right] \Rightarrow A_t = 2.55*10^{-4}\;m^2$$

$\Rightarrow$ the diameter of throat dt = 18 mm
At exit (P_2/P_1) = 0.7/3.5 = 0.2

$$A_t^2 = (1.3)^2\left(\frac{0.0633}{3.5*10^6}\right)\left[\frac{(0.2)^{-\frac{2}{1.4}}}{\left[1-(0.2)^{\frac{0.4}{1.4}}\right]}\right] \Rightarrow A_t = 3.436*10^{-4}\;m^2$$

$\Rightarrow$ the diameter of exit region dE = 21 mm

Or another method

$$u_t = u_w = \sqrt{\gamma P_t v_t} \qquad P_t = 1.85\,MPa \quad v_t = v_1 (P_t/P_1)^{\frac{-1}{\gamma}} = 0.0633(0.528)^{\frac{-1}{1.4}} = 0.0999\,m^3/kg$$

$$\Rightarrow u_t = \sqrt{1.4(1.85*10^6)(0.0999)}$$
$$= 508.666\,m/s \quad (sonic\,velocity)$$

$$A_t = \dot{m}\frac{v_t}{u_t} = 1.3(0.0999/508.666) = 2.55*10^{-4} m^2$$

Or another method

$$u_2^2 = \left(\frac{2\gamma}{\gamma-1}\right) P_1 v_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right] = 1,550,850\left[1 - \left(\frac{P_2}{P_1}\right)^{0.2857}\right]$$

$$u_t^2 = 258671.997 \Rightarrow u_t = 508.6\ m/s \qquad u_2^2 = 571666.52 \Rightarrow u_2 = 756.086\ m/s$$

$$v_2 = v_1\left(\frac{P_2}{P_1}\right)^{-\frac{1}{\gamma}} = 0.0633(0.2)^{\frac{-1}{1.4}} = 0.1998\ m^3/kg$$

$$A_2 = \dot{m}\frac{v_2}{u_2} = 1.3(0.198/756.086) = 3.3436*10^{-4}\,m^2$$

8.5 Flow Measurement for Compressible Fluid

For horizontal flow with no shaft work and neglecting the frictional energy tem, the net of the general energy will be: -

$$\frac{u_2^2}{2\alpha_2} - \frac{u_1^2}{2\alpha_1} + \int_{P_1}^{P_2} v\,dp = 0 \quad but \;\; \dot{m}_1 = \dot{m}_2 = \dot{m} \Rightarrow u_1 = \frac{v_1}{v_2}\frac{A_2}{A_1}u_2$$

- **For isothermal flow**

$$\int_{P_1}^{P_2} v\,dp = P_1 v_1 \ln\frac{P_2}{P_1} \qquad \Rightarrow u_2^2 - \left(\frac{v_1}{v_2}\frac{A_2}{A_1}u_2\right)^2\frac{\alpha_2}{\alpha_1} + 2\alpha_2 P_1 v_1 \ln\frac{P_2}{P_1} = 0$$

$$\Rightarrow u_2^2 = \frac{2\alpha_2 P_1 v_1 \ln(P_2/P_1)}{1 - \frac{\alpha_2}{\alpha_1}\left(\frac{v_1}{v_2}\frac{A_2}{A_1}\right)^2} ----(1)$$

- **For adiabatic flow**

$$v = v_1 P_1^{\frac{1}{\gamma}} P^{\frac{1}{\gamma}} \qquad \Rightarrow u_2^2 = \frac{2\alpha_2 P_1 v_1\left(\frac{\gamma}{\gamma-1}\right)\left|\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right|}{1 - \frac{\alpha_2}{\alpha_1}\left(\frac{v_1}{v_2}\frac{A_2}{A_1}\right)^2} ---(2)$$

It should be noted that equations (1) and (2) apply provided that (P_2/P_1) is greater than the critical pressure ratio (r_c). Where if $(P_2/P_1) < (r_c)$, the flow becomes independent on P_2 and conditions of maximum flow occur.

8.6 Fans, Blowers, and Compression Equipment

Fans and blowers are used for many types of ventilating work such as air-conditioning systems. In large buildings, blowers are often used due to the high delivery pressure needed to overcome the pressure drop in the ventilation system.

Blowers are also used to supply draft air to boilers and furnaces.

Fans are used to move large volumes of air or gas through ducts, supplying air to drying, conveying material suspended in the gas stream, removing fumes, condensing towers and other high flow, low pressure applications.

Fans are used for low pressure where generally the delivery pressure is less than 3.447 kPa (0.5 psi), and blowers are used for higher pressures. However they are usually below delivery pressure of 10.32 kPa (1.5 psi). These units can either be **centrifugal** or the **axial-flow** type.

The axial flow type in which the air or gas enters in an axial direction and leaves in an axial direction.

The centrifugal blowers in which the air or gas enters in the axial direction and being discharge in the radial direction.

Compressors

Compressor are used to handle large volume of gas at pressures increase from 10.32 kPa (1.5 psi) to several hundred kPa or (psi). Compressors are classified into: -

1- Cotinuous-flow compressors

1-a- Centrifugal compressors

1-b- Axial-flow compressors

2- Positive displacement compressors

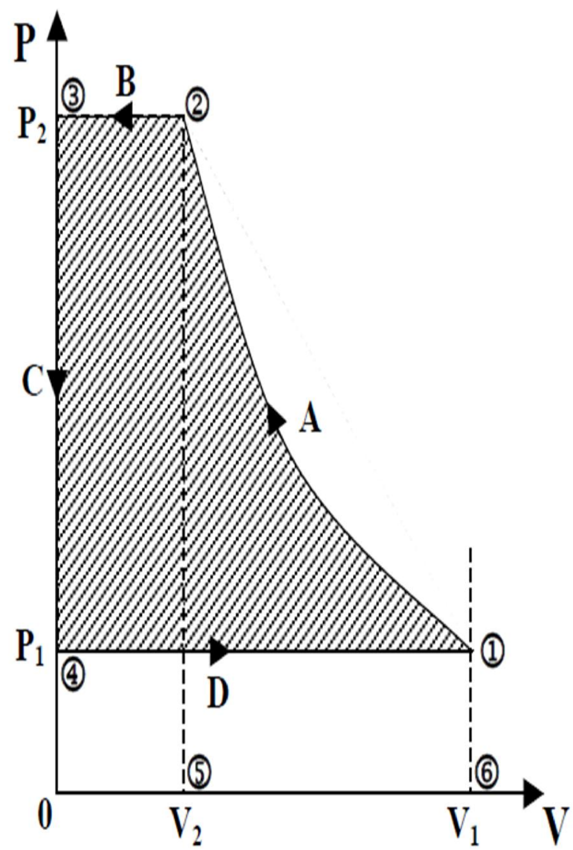
2-a- Rotary compressors

2-b- Reciprocating compressors

Since a large proportion of the energy of compression appears as heat in the gas, there will normally be a considerable increase in temperature, which may limit the operation of the compressors unless suitable cooling can be effected. For this reason gas compression is often carried out in a number of stages and the gas is cooled between each stage.

8.7 <u>Gas Compression Cycle</u>

Suppose that, after the compression of a volume V1 of gas at P1 to a pressure P2, the whole of the gas is expelled at constant pressure P2, and a fresh charge of gas is admitted at a pressure P1. The cycle can be followed as in Figure, where **P is plotted as <u>ordinate against V as abscissa.</u>**



Point ❶ represents the initial conditions of the gas of pressure and volume of (P_1,V_1).

► A-line ❶→❷ **Compression of gas from (P_1,V_1) to (P_2,V_2).**

► B-line ❷→❸ **Expulsion of gas at constant pressure P_2.**

► C-line ❸→❹ **Sudden reduction in pressure in the cylinder from P_2 to P_1. As the whole of the gas has been expelled.**

► D-line ❹→ ❶ **A fresh charge of the gas through the suction stroke of the piston, during which a volume V_1 of gas is admitted at constant pressure P_1.**

The Total Work Done Per Cycle

It will be noted that the mass of gas in the cylinder varies during the cycle. The work done by the compressor during each of the cycle is as follows: -

-step (A): Compressor	$-\int_{V_1}^{V_2} P\,dV$	**[area ❶→❷→❺→❻]**
-step (B): Expulsion	P_2V_2	**[area ❷→❸→⓿→❺]**
-step (D): Suction	$-P_1V_1$	**[area ❹→⓿→❻→❶]**

∴ the total work done per cycle = $-\int_{V_1}^{V_2} P\,dV + P_2V_2 - P_1V_1$ **[area ❶→❷→❸→❹]**

dPV = P dV + V dP ⇒ P dV = dPV – V dP

$$-\int_{V_1}^{V_2} P\,dV = \int_{P_1}^{P_2} V\,dp - \int_{P_1V_1}^{P_2V_2} dPV$$ **but PV = RT and dPV = R dT**

$$\Rightarrow \int_{P_1V_1}^{P_2V_2} dPV = R\int_{T_1}^{T_2} dT = RT_2 - RT_1 = (P_2V_2 - P_1V_1)$$

$$\Rightarrow -\int_{V_1}^{V_2} P\,dV = \int_{P_1}^{P_2} V\,dp - (P_2V_2 - P_1V_1)$$

$$= \int_{P_1}^{P_2} V\,dp - \cancel{P_2V_2} + \cancel{P_1V_1} + \cancel{P_2V_2} - \cancel{P_1V_1} = \int_{P_1}^{P_2} V\,dp$$

Or The total work done per cycle (W)= $-\int_{V_1}^{V_2} P\,dV + \Delta PV$

$$\Rightarrow dW = -PdV + dPV = -Pdv + VdP + PdV \quad \Rightarrow dW = dPV \quad \Rightarrow \; W = \int_{P_1}^{P_2} V\,dp$$

- **Under isothermal conditions**

The work of compression for an ideal gas per cycle= $\int_{P_1}^{P_2} V\,dp = RT\int_{P_1}^{P_2} dP/P = RT\,ln(P_2/P_1)$

- **Under adiabatic conditions**

The work of compression for an ideal gas per cycle $= \int_{P_1}^{P_2} V\,dp = V_1 P_1^{\frac{1}{\gamma}} \int_{P_1}^{P_2} P^{\frac{1}{\gamma}}\,dP$

$$- P_1V_1\frac{\gamma}{(\gamma-1)}\left[(P_2/P_1)^{((\gamma-1)/\gamma)} - 1\right]$$

8.7.1 Clearance Volume

It practice, it is not possible to expel the whole of the gas fro the cylinder at the end of the compression; the volume remaining in the cylinder after the forward stroke of the piston is termed "**the clearance volume".**

The volume displaced by the piston is termed "**the swept volume", and therefore the total volume of the cylinder is made up of the clearance volume plus the swept volume.**

i.e. Total volume of cylinder = [clearance volume + swept volume]

A typical cycle for a compressor with a finite clearance volume can be followed by reference to the Figure;

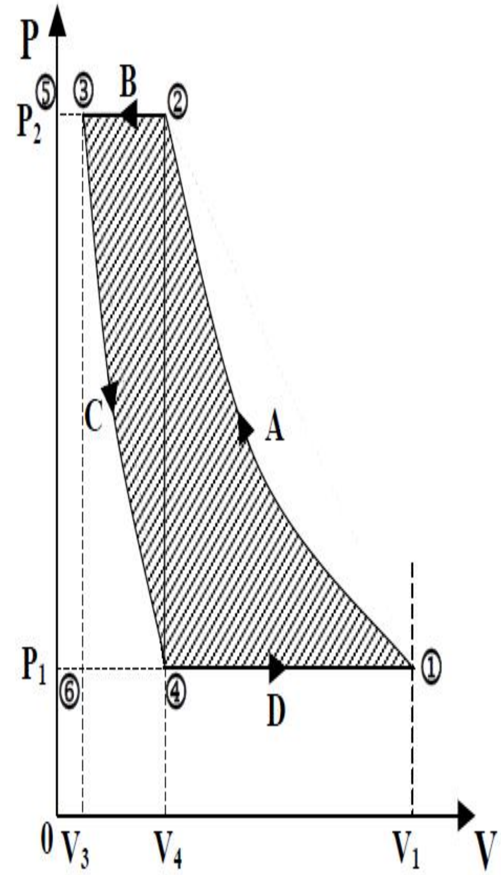
volume V_1 of gas at a pressure P_1 is admitted to the cylinder; its condition is represented by point $_1$,

► **A-line ❶→❷ Compression of gas from (P_1,V_1) to (P_2,V_2).**

► **B-line ❷→❸ Expulsion of gas at constant pressure P2, so that the volume remaining in the cylinder is V_3.**

► **C-line ❸→❹ Expansion of this residual gas to the lower pressure P1 and volume V_4 during the return stroke.**

► **D-line ❹→❶ Introduction of fresh gas into the** cylinder at constant pressure **P.**



Total Work Done Per Cycle

-step (A): Compressor $-\int_{V_1}^{V_2} V\,dp$

-step (A): Expulsion $P_2(V_2 - V_3)$

-step (A): Expansion $-\int_{V_3}^{V_4} V\,dp$

-step (A): Suction $-P_1(V_1 - V_4)$

The total work done per cycle is equal to the sum of these four components. It is represented by by the selected area [i.e. area ❶→❷→❸→❹], **which is equal to** [area❶→❷→❺→❻] **less [area ❸→❹→❺→❻]**

● Under isentropic conditions

The work done per cycle

$$= \int_{P_1}^{P_2} V\,dp - \int_{P_4}^{P_3} V\,dp = \frac{\gamma}{\gamma-1} P_1V_1\left[(P_2/P_1)^{((\gamma-1)/\gamma)}-1\right] - \frac{\gamma}{\gamma-1} P_4V_4\left[(P_3/P_4)^{((\gamma-1)/\gamma)}-1\right]$$

But (P_1=P_4) and (P_2=P_3) $\Rightarrow W = \frac{\gamma}{\gamma-1} P_1(V_1-V_4)\left[\left(\frac{P_2}{P_1}\right)^{((\gamma-1)/\gamma)}-1\right]$

Now V is not know explicitly, but can be calculated in terms of V_3, the clearance volume, for isentropic conditions

$$\mathbf{V_4 = V_3\,(P_2/P_1)^{1/\gamma}}$$

And $\mathbf{V_1 - V_4 = (V_1 - V_3) + V_3 - V_3(P_2/P_1)^{1/\gamma}}$

$$\mathbf{= (V_1 - V_3)\,[1 + \{V_3/(V_1 - V_3)\} - \{V_3/(V_1 - V_3)\}\,(P_2/P_1)^{1/\gamma}]}$$

Where

$\mathbf{(V_1 - V_3) = V}$: the swept volume

$\mathbf{V_3}$: the clearance volume

$\mathbf{V_3/(V_1 - V_3) = C}$: the clearance

$\Rightarrow\ V_1 - V_4 = V_s\,[1+C-C(P_2/P_1)^{1/\gamma}]$

∴ The total work done on the fluid per cycle is there for:-

$$W = \frac{\gamma}{\gamma-1} P_1 V_s \left[\left(\frac{P_2}{P_1}\right)^{\left(\frac{(\gamma-1)}{\gamma}\right)} - 1\right]\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]$$

The factor $\left[1 + c - c\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right]$ is called

" the theoretical volumetric efficiency", and is a measure of the effect of the clearance on an isentropic compression. Cooling during compression make the work down per cycle less than that given by the latest equation, (γ) is replaced by smaller quantity(k).

The greater the rate of heat removal, the less in the work down.

Notice that the isothermal compression is usually taken as the condition for the least work of compression. The actual work of compression is greater than the theoretical work because of clearance gases , back leakage, and frictional effects, where, $\eta = W_{theo.}/W_{act.}$

8.8 Multistage Compressors

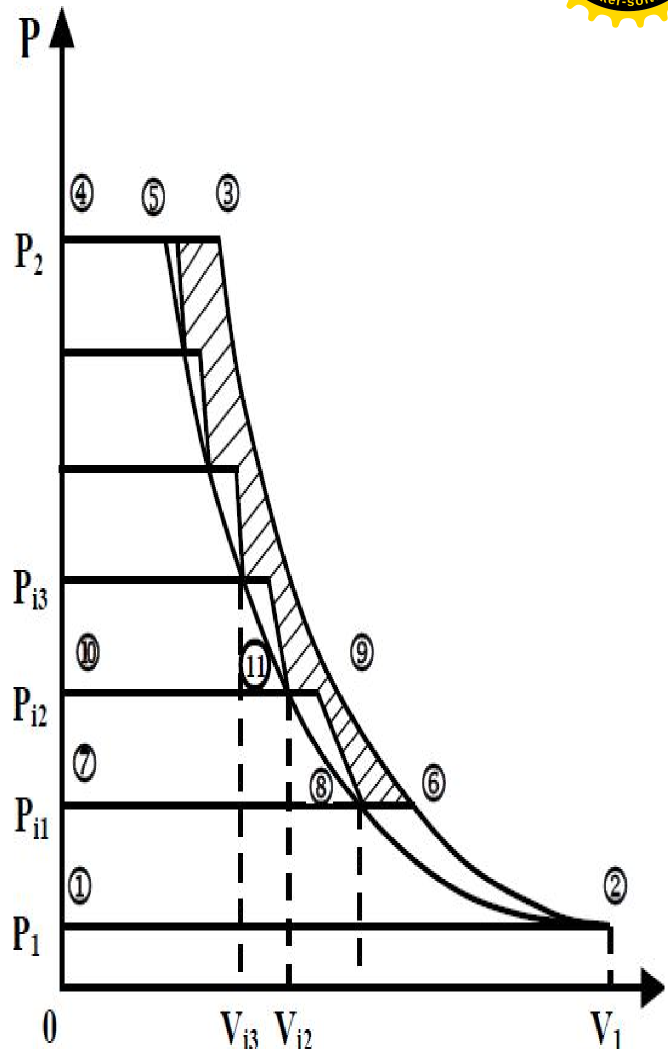
The maximum pressure ratio normally in a single cylinder is (10) but values above (6) are usual. If the required pressure ratio (P_2/P_1) is large, it is not practicable to carry out the whole of the compression in a single cylinder because of the high temperatures, which would be set up, and the adverse effects of clearance volume on the efficiency. Further, lubrication would be difficult due to carbonization of the oil and there would be a risk of causing oil mist explosions in the cylinders when gases containing oxygen were being compressed. The operation of the multistage compressor can conveniently be followed again on a pressure-volume diagram as shown in the Figure,

The area [❶→❷→❸→❹]**represents the work done in compression isentropically from P_1 to P_2 in a single stage.**

The [area❶→❷→❺→❹❻] **represents the necessary work for an isothermal compression.**

Now consider a multistage isentropic compression in which the intermediate pressures are P_{i1}, P_{i2}, P_{i3}, …………etc.

The gas will be assumed to be cooled to its initial temperature in an inter-stage cooler before it enters each cylinder.



► A-line ❶→❷ represents the suction storke of the first stage where a volume (V_1) of gas is admitted at a pressure(P_1)

► B-line ❷→ ❻ represents isentropic compression to a pressure(P_{i1})

► C-line ❻→❼ represents the delivery of the gas from the first stage at a constant pressure(P_{i1}).

► D-line ❼→❽ represents the suction storke of the second stage. The volume of the gas has reduced in the inter-stage cooler to (V_{i1}), that which would have been obtained as a result of an isothermal compression to (P_{i2}).

►**E**-line ❽→❾ represents an isentropic compression in the second stage from a pressure (Pi1) to a pressure (Pi2).

►**F**-line ❾→❿ represents the delivery stroke of the second stage.

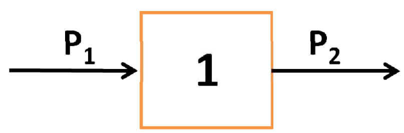
►**G**-line ❿→⓫represents the suction stroke of the third, point⓫ again lyses on the line ❷→❺ that representing an isothermal compression.

It seen that the overall work done on the gas is intermediate between that for a single stage isothermal compression and that for isentropic compression. The net saving in energy is shown as the shaded area in the last Figure.

The Total Work Done for Multistage Compressors

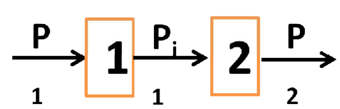
- The total work done for compression the gas from P_1 to P_2 in an ideal single stage is,

$$W = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\left(\frac{(\gamma - 1)}{\gamma} \right)} - 1 \right]$$



- The total work done for compression the gas from P_1 to P_2 in an ideal two stages is,

$$W = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[\left(\frac{P_{i1}}{P_1} \right)^{\left(\frac{(\gamma-1)}{\gamma} \right)} - 1 \right] + \frac{\gamma}{\gamma - 1} P_{i1} V_{i1} \left[\left(\frac{P_2}{P_{i1}} \right)^{\left(\frac{(\gamma-1)}{\gamma} \right)} - 1 \right]$$



but for perfect inter-stage cooling i.e. at isothermal line

$$\mathbf{P_1 V_1 = P_{i1} V_{i1} = constant}$$

$$\Rightarrow W = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[\left\{ \left(\frac{P_{i1}}{P_1} \right)^{\left(\frac{(\gamma - 1)}{\gamma} \right)} - 1 \right\} + \left\{ \left(\frac{P_2}{P_{i1}} \right)^{\left(\frac{(\gamma - 1)}{\gamma} \right)} - 1 \right\} \right]$$

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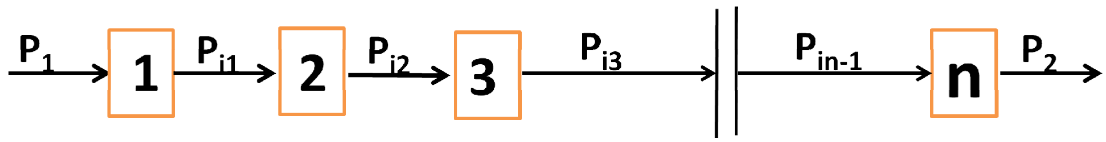
● The total work done for compression the gas from P_1 to P_2 in an ideal n-stages is,

$$\Rightarrow W=\frac{\gamma}{\gamma-1}P_1V_1\left[\left(\frac{P_{i1}}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1\right]+\frac{\gamma}{\gamma-1}P_{i1}V_{i1}\left[\left(\frac{P_{i2}}{P_{i1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1\right]+\cdots+\frac{\gamma}{\gamma-1}P_{in-1}V_{in1}\left[\left(\frac{P_2}{P_{in-1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1\right]$$



for perfect inter-stage cooling

$P_1V_1= P_{i1}V_{i1}= P_{i2}V_{i2}=$ ------ $= P_{in-1}V_{in-1}=$ constant

$$\Rightarrow W=\frac{\gamma}{\gamma-1}P_1V_1\left[\left\{\left(\frac{P_{i1}}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1\right\}+\left\{\left(\frac{P_{i2}}{P_{i1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1\right\}+\cdots+\left\{\left(\frac{P_2}{P_{in1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1\right\}\right]$$

$$\Rightarrow W=\frac{\gamma}{\gamma-1}P_1V_1\left[\left(\frac{P_{i1}}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}+\left(\frac{P_{i2}}{P_{i1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}+\cdots+\left(\frac{P_2}{P_{in1}}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-n\right]$$

The optimum values of intermediate pressures P_{i1}, P_{i2}, P_{i3}, -------P_{in-1} are so that **The compression ratio (r) is the same in each stage and equal work is then done in each stage.**

$$i.e.\frac{P_{i1}}{P_1}=\frac{P_{i2}}{P_{i1}}=\frac{P_{i3}}{P_{i2}}=\cdots\ldots\ldots\frac{P_2}{P_{in1}}=r$$

$$then\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}=\frac{P_{i1}}{P_1}=\frac{P_{i2}}{P_{i1}}=\frac{P_{i3}}{P_{i2}}=\cdots\frac{P_2}{P_{in1}}=r---prove\ that$$

$$\Rightarrow W = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[n \left(\frac{P_2}{P_1} \right)^{\left(\frac{(\gamma-1)}{\gamma}\right)} - n \right] \qquad \Rightarrow W = P_1 V_1 \frac{n\gamma}{\gamma - 1} \left| \left(\frac{P_2}{P_1} \right)^{\left(\frac{(\gamma-1)}{\gamma}\right)} \quad n \right|$$

The effect of clearance volume can now be taken into account. If the clearance in the successive cylinder are C_1, C_2, C_3,….C_n the theoritical volumetric efficiency of the first cylinder= $[1+ C_1 - C_1(Pi1 /P_1)^{1/\gamma}]$.

Assuming that The same compression ratio is used in each cylinder, then the theoretical volumetric efficiency of the first stage

$=[1+ C_1 - C_1(Pi1 /P_1)^{1/n\gamma}]$.

If the swept volume of the cylinders are V_{s1}, V_{s2}, V_{s3} ,……… the volume of gas admitted to the first cylinder

$= V_{s1} =[1+ C_1 - C_1(Pi1 /P_1)^{1/n\gamma}]$.

The same mass of gas passes through each of the cylinders and, therefore, if the inter-stage coolers are assumed perfectly efficient, the ratio of the volumes of gas admitted to successive cylinder is $(P1 /P_2)^{1/n}$ [because lies on the isothermal line]. The volume of the gas admitted to the cylinder

$$= Vs_2 \,[1 + C_2 - C_2(P_2 /P_1)^{1/n\gamma}] = Vs_1 \,[1 + C_1 - C_1(P_2 /P_1)^{1/n\gamma}] (P_1 /P_2)^{\frac{1}{n}}$$

$$\Rightarrow \frac{Vs_2 \,[1 + C_2 - C_2(P_{i1} /P_1)^{1/n\gamma}]}{Vs_1 \,[1 + C_1 - C_1(P_2 /P_1)^{1/n\gamma}]} \quad (P_2 /P_1)^{\frac{1}{n}}$$

In this manner the swept volume of each cylinder can be calculated in terms of V_{s1}, and C_1, C_2, …………., and the cylinder dimensions determined.

$$\text{Let } V_1 = Vs_1 \,[1 + C_1 - C_1(P_2 /P_1)^{1/n\gamma}] \;, V_2 = Vs_2 \,[1 + C_2 - C_2(P_2 /P_1)^{1/n\gamma}]$$

Where , V_i: represent the volume of gas admitted to stage i. But for perfectly cooled[i.e. isothermal] $\Rightarrow P_iV_1 = P_{i1}V_1 = P_{i2}V_2 = \text{------} = P_{in-1}V_n$

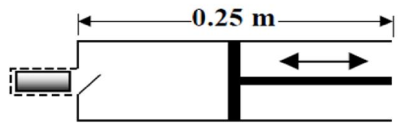
$$\Rightarrow P_1 Vs_1\left[1 + C_1 - C_1\left(\frac{P_2}{P_1}\right)^{\frac{1}{n\gamma}}\right] = P_{i1}Vs_2\left[1 + C_2 - C_2\left(\frac{P_2}{P_1}\right)^{1/n\gamma}\right]$$

$$\text{But } r = \frac{P_{i1}}{P_1} = \left(\frac{P_2}{P_i}\right)^{\frac{1}{n}}$$

$$\Rightarrow \frac{Vs_1}{Vs_2} = \frac{\left[1 + C_2 - C_2\left(\frac{P_2}{P_1}\right)^{\frac{1}{n\gamma}}\right]}{\left[1 + C_1 - C_1\left(\frac{P_2}{P_1}\right)^{\frac{1}{n\gamma}}\right]}\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

Example -8.12-

A single-acting air compressor supplies 0.1 m^3/s of air (at STP) compressed to 380 kPa from 101.3 kPa. If the suction temperature is 289 K, the stroke is 0.25 m, and the speed is 4 Hz, what is the cylinder diameter? Assume the cylinder clearance is 4% and compression and re-expansion are isentropic (γ=1.4). What are the theoretical power requirements for the compression?



Solution:

Stroke ("حركة من سلسلة حركات متسلسلة "متوالية ومتشابهة)

Volume of gas per stroke = $(0.1\ m^3/s)/4s^{-1}\ (289/273)$

$= 0.0264\ m^3$

$=(V_1-V_4) \equiv$[volume of gas admitted per cycle]

$P_2/P_1 = 380/101.3 = 3.75$

$(V_1-V_4) = V_s\ [1+ C - C\ (P_2/P_1)^{1/\gamma}]$

$0.0264 = V_s[1+0.04-0.04(3.75)^{1/1.4}] \Rightarrow V_s = 0.0283\ m^3 = (V_1 - V_3) \equiv$ volume of cylinder

Cross-section area of cylinder = $V_s/L_{stroke} = 0.0283/0.25 = 0.113\ m^2$

$\Rightarrow$ The diameter of cylinder = $[0.113/(\pi/4)]^{1/2}$ =0.38 m

$$W = \frac{\gamma}{\gamma-1}P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

for 1kg of gas that compressed [or per cycle]

$$\Rightarrow W = \frac{1.4}{0.4}(101.3\times10^3)(0.0264)\left[(3.75)^{0.4/1.4} - 1\right] = 4278\ J/kg \text{ per stroke}$$

The theoretical power required = 4278 J/kg $(4s^{-1})$ per stroke= 17110 W =17.11kW

Example -8.13-

Air at 290 K is compressed from 101.3 kPa to 2065 kPa in two-stage compressor operating with a mechanical efficiency of 85%. The relation between pressure and volume during the compression stroke and expansion of clearance gas is ($PV^{1.25}$ = constant). The compression ratio in each of the two cylinders is the same, and the inter-stage cooler may be assumed 100% efficient. If the clearance in the two cylinders are 4% and 5%, calculate:

a- The work of compression per kg of air compressed;
b- The isothermal efficiency;
c- The isentropic efficiency;
d- The ratio of swept volumes in the two cylinders.

Solution:

$P_2/P_1 = 2065/101.3 = 20.4$

$$V_1 = \frac{RT}{P_1 Mwt} = \frac{8314\ (\text{Pa.m}^3/\text{kmol.K})\ 290\text{K}}{(101.3\times10^3\ Pa)29\ \text{kg/kmol}} = 0.82 \quad (m^3/kg)$$

For 100% efficient of cooler at inter-stage, the work of compression in multistage compressor of n-stages is;

$$W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma}-1\right] \Rightarrow W = (101.3\times10^3\times0.82)\frac{2(1.25)}{1.25-1}\left[(20.4)^{(0.25)/2.5}-1\right] = 292.35\frac{kJ}{kg}$$

The work of compressor = $W_{act} = W/\eta = 292.3/0.85 = 344$ kJ/kg

For isothermal compression = $W_{iso} = P_1V_1 \ln(P_2/P_1) = 250.5$ kJ/kg

Isothermal efficiency = $(W_{iso}/W_{act})\ 100 = 72.8$ %

For isentropic compression = $W_{adb} = P_1V_1\ \gamma/(\gamma-1)\ [(P_2/P_1)^{(\gamma-1)/\gamma} - 1] = 397.4$ kJ/kg

Isentropic efficiency = $(W_{adb}/W_{act})\ 100 = 115.5$ %

$$V_1 = V_{s1}\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]$$

$$\Rightarrow 0.82 = V_{s1}\left[1 + 0.04 - 0.04(20.4)^{1/2.5}\right] \Rightarrow V_{s1} = 0.905\ m^3/kg$$

The swept volume of the second cylinder is given by:

$$V_{s2} = V_{s1}\frac{\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]}{\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]}\left(\frac{P_1}{P_2}\right)^{1/n} \longleftarrow \text{انتبه}$$

$$V_{s2} = \frac{V_1(P_1/P_2)^{1/n}}{\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]} = \frac{0.82(1/20.4)^{1/2}}{\left[1 + 0.05 - 0.05(20.4)^{1/2.5}\right]} = 0.206\ m^3/kg$$

$\therefore \quad V_{s1}/V_{s2} = 0.905/0.206 = 4.4$

Example -8.14-

Calculate the theoretical work in (J/kg) required to compress a diatomic gas initially at T = 200 K adiabatically compressed from a pressure of 10 kPa to 100 kPa in;

1- Single stage compressor;
2- Two equal stages;
3- Three equal stages; Taken that $\gamma = 1.4$, Mwt = 28 kg/kmol

Solution:

1- $$W = P_1V_1 \frac{\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

$P_2/P_1 = 100/10 = 10$

$$V_1 = \frac{RT}{P_1\,Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\,200\text{K}}{(10\times10^3\,Pa)28\ \text{kg/kmol}} = 5.94 \quad (m^3/kg)$$

$$\Rightarrow W = 10(5.92)\frac{1.4}{0.4}\left[(10)^{0.4/1.4} - 1\right] = 193.44\ kJ/kg$$

2- $$W = P_1V_1 \frac{n\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = 59.4\frac{2(1.4)}{0.4}\left[(10)^{(0.4)/2.8} - 1\right] = 161.95\frac{kJ}{kg}$$

3- $$W = P_1V_1 \frac{n\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = 59.4\frac{3(1.4)}{0.4}\left[(10)^{(0.4)/4.2} - 1\right] = 152.93\frac{kJ}{kg}$$

For 100% efficient of cooler at inter-stage, the work of compression in multistage compressor of n-stages is;

$$W = P_1V_1 \frac{n\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = (101.3\times10^3\times0.82)\frac{2(1.25)}{1.25-1}\left[(20.4)^{(0.25)/2.5} - 1\right] = 292.35\frac{kJ}{kg}$$

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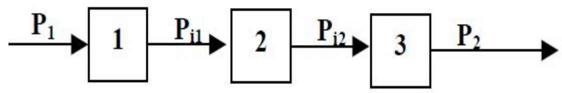
Example -8.15-

A three stages compressor is required to compress air from 140 kPa and 283 K to 4000 kPa. Calculate the ideal intermediate pressures, the work required per kg of gas, and the isothermal efficiency of the process. Assume the compression to be adiabatic and perfect the inter-stage cooling to cool the air to the initial temperature. Taken that $\gamma = 1.4$.

Solution:

$$\frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} = \frac{P_2}{P_{i2}} = r = \left(\frac{P_2}{P_1}\right)^{\frac{1}{3}} = \left(\frac{4000}{140}\right)^{\frac{1}{3}} = 3.057$$



$\Rightarrow$ P_{i1}=3.057 (140) =428 kPa

P_{i2}=3.057 (428) =1308.4 kPa

$$W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right]$$

$$P_1V_1 = \frac{RT}{Mwt} = \frac{8314\ (\text{Pa.m}^3/\text{kmol.K})\ 283\text{K}}{(29\ \text{kg/kmol})} = 81.133 \quad (kJ/kg)$$

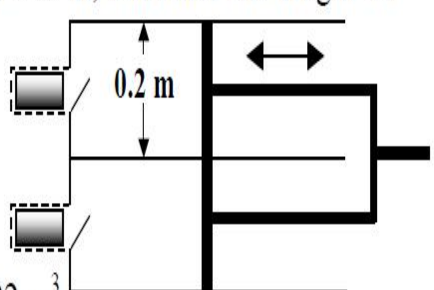
$$\Rightarrow W = 81.133\frac{3(1.4)}{0.4}\left[\left(\frac{4000}{140}\right)^{0.4/4.2} - 1\right] = 320.43\ kJ/kg$$

For isothermal compression = $W_{iso} = P_1V_1 \ln(P_2/P_1) = 272$ kJ/kg

Isothermal efficiency = $(W_{iso}/\ W)\ 100 = 84.88\ \%$

Example -8.16-

A twin-cylinder single-acting compressor, working at 5 Hz, delivers air at 515 kPa pressure at the rate of 0.2 m^3/s. If the diameter of the cylinder is 20 cm, the cylinder clearance ratio 5%, and the temperature of the inlet air 283 K, calculate the length of stroke of the piston and delivery temperature (γ=1.4).



Solution:

$T_2/T_1 = (P_2/P_1)^{(\gamma-1)/\gamma}$ $\Rightarrow$ T_2=283(515/101.3)$^{0.4/1.4}$=450K

The volume handled per cylinder = 0.2/2 =0.1 m^3/s

Volume per stroke per cylinder = (0.1 m^3/s) / (5 s^{-1}) = 0.02 m^3

Volume at inlet conditions = (0.02 m^3) (283/450) (515/101.3) = 0.0639 m^3

$$V_1 - V_4 = V_s\,[1 + C - C\,(P_2/P_1)^{1/\gamma}] \Rightarrow\ 0.0639 = V_s\,[1 + 0.05 - 0.05\,(515/101.3)^{1/1.4}]$$

$$\Rightarrow\quad V_s = 0.0718\ m^3 = \pi/4\ (0.2)^2\ L_{stroke} \Rightarrow\quad L_{stroke} = 2.286\ m$$

Example -8.17-

In a single-acting compressor suction pressure and temperature are 101.3 kPa and 283 K, the final pressure is 380 kPa. If the compression is adiabatic and each new charge is heated 18 K by contact with the clearance gases, calculate the maximum temperature attained in the cylinder (γ=1.4).

Solution: On the first stroke the air enters at 283 K and is compressed adiabatically

$\Rightarrow \quad T_2 = 283\,(380/101.3)^{0.4/1.4} = 415\text{ K}$

The clearance volume gases at 413 K which remain in the cylinder are able to raise the next cylinder full of air by 18 K i.e. the air temperature in the next cylinder is [18 + 283 = 301 K] $\Rightarrow$ The exit temperature = $301\,(380/101.3)^{0.4/1.4} = 439.2$ K

On each subsequent stroke T_{in}=283 K, $T_{cylinder}$ = 301 K, and T_{exit} = 439.2 K.

Example -8.18-

A single-stage double-acting compressor running at 3 Hz is used to compess air from 110 kPa and 282 K to 1150 kPa. If the internal diameter of the cylinder 20 cm, the length of the stroke 25 cm, and the piston clearance 5%. Calculate;

a- The maximum capacity of machine, referred to air at initial conditions;

b- The theoretical power requirements under isentropic conditions.

Solution:

The swept volume per stroke = $2[\pi/4\,(0.2)^2\,(0.25)] = 0.0157\text{ m}^3$

$(V_1 - V_4) = V_s[1 + C - C\,(P_2/P_1)^{1/\gamma}] \Rightarrow (V_1 - V_4) = 0.0157[1 + 0.05 - 0.05\,(1150/110)^{1/1.4}]$

$\Rightarrow \quad (V_1 - V_4) = 0.0123\text{ m}^3$

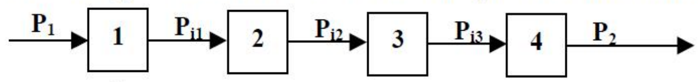
$$W = P_1(V_1 - V_4)\frac{\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] \Rightarrow W = 110(0.0123)\frac{1.4}{0.4}\left[\left(\frac{1150}{110}\right)^{0.4/1.4} - 1\right] = 5.775\ kJ/stroke$$

The power required = (3 stroke/s)(5.775 kJ/stroke) = 17.324 kW

Capacity = (3 stroke/s) (0.0123 m^3/stroke) = 0.0369 m^3/s

Example -8.19-

Methane is to be compressed fom atmospheric pressure ton 30 MPa in four stages. Calculate the ideal intermediate pressures and the work required per kg of gas. Assume compression to be isentropic and the gas to behave as an ideal gas and the initial condition at STP (γ=1.4).



Solution:

$$\frac{P_{i1}}{P_1}=\frac{P_{i2}}{P_{i1}}=\frac{P_{i3}}{P_{i2}}=\frac{P_2}{P_{i3}}=r=\left(\frac{P_2}{P_1}\right)^{\frac{1}{4}}=\left(\frac{30}{0.1013}\right)^{\frac{1}{4}}=4.148$$

$\Rightarrow$ P_{i1}=4.148 (101.3 kPa) = 420.23 kPa

P_{i2}=4.148 (420.23 kPa) = 1743.27 kPa

P_{i3}=4.148 (1743.27 kPa) = 7231.75 kPa

P_2=4.148 (7231.75 kPa) = 30,000 kPa

$$W=P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma}-1\right]$$

$$(P_1V_1)_{STP}=\frac{RT}{Mwt}=\frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 273\text{K}}{(16\ \text{kg/kmol})}=141.857\quad (kJ/kg)$$

$$\Rightarrow W=141.857\frac{4(1.4)}{0.4}\left[\left(\frac{30{,}000}{101.3}\right)^{0.4/5.6}-1\right]=996.06\ kJ/kg$$